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Combination tones – intonation companions

Over 300 years ago, Giuseppe Tartini recognised that intervals could be measured by means of a combination tone which he called *terzo suono* (third tone). When two tones are played in their exact vibration ratio, a third tone (or even more additional tones) appears at a mathematically predictable position. This acoustic phenomenon is the basis for manifold applications which awaken the ear, increase its sensitivity, and generally expand the notion of sound. On the violin, for example, syntonic scales (diatonic or chromatic) can be created without external technical aids—only using the ear. Of special interest is the lowest combination tone, which forms a real-sounding bass accompaniment. Musical examples show how this demands from the performer clear decisions about intonation and the size of specific intervals. An awareness of the phenomenon of combination tones adds a new dimension to the interpretation of works such as J. S. Bach's Sonatas and Partitas for solo violin, and helps to navigate intonation in ensemble playing.

Bereits vor 300 Jahren stellte Giuseppe Tartini fest, dass Intervalle mithilfe eines Kombinationstons, des *terzo suono* (dritten Tons), exakt gemessen werden können. Erklingen zwei Töne gleichzeitig in ihrem exakten Schwingungsverhältnis, erscheint ein dritter Ton oder es erscheinen sogar mehrere mitschwingende Töne ebenfalls an ihrer mathematisch voraussagbaren Stelle. Diese Naturgegebenheit ist die Basis für vielfältige Anwendungsmöglichkeiten, die das Gehör sensibilisieren können und den Klangbegriff erweitern. Auf der Violine beispielsweise, können syntonische Skalen (diatonische oder chromatische) mithilfe der Kombinationstöne ohne zusätzliche, technische Hilfen, allein über das Gehör abgemessen und umgesetzt werden. Von besonderem Interesse ist der unterste Kombinationston, der eine klingende Bassbegleitung darstellt. Die musikalischen Beispiele in diesem Artikel zeigen, wie dies von der Spielerin/vom Spieler genutzt werden kann und welche Entscheidungen dies hinsichtlich der Intonation spezifischer Intervalle erfordert. Das Wissen über das Phänomen bringt eine neue Interpretationsdimension in Werke wie J. S. Bachs Sonaten und Partiten für Violine solo und hilft der Intonationsnavigation im Ensemblespiel.

SCHLAGWORTE/KEYWORDS: combination tones; Giuseppe Tartini; instrumental practice; Instrumentalpraxis; intonation; Intonation; just intervals; Kombinationstöne; reine Intervalle; violin; Violine

Introduction

The knowledge of combination tones sharpens a musician's ear and allows access to a new and subtle realm of sound. Hearing awareness expands and pitch discrimination increases, setting in motion a process which cannot be undone and which offers advanced understanding of the correlation between tone and number.

In the second part of this article (see section “Benefits for instrumental practice”) it will be demonstrated that the informed and adroit use of combination tones can refine the interpretation of works such as Bach’s Sonatas and Partitas for solo violin or Vivaldi’s and Tartini’s compositions for string instruments.

The phenomenon

When two notes are played simultaneously, additional tones at a lower volume can often be perceived. These so-called ‘combination tones’[1] may appear below and above the played notes, although lower combination tones are normally more distinct and easier to detect. The phenomenon becomes particularly apparent with intervals of simple integer frequency ratios, and with stable and sustained sound. The notes do not need to be played loudly: in fact, experience shows that combination tones often become clearer and more distinct when the tones are played softly.[2] Though combination tones are generated by a neurophysiological mechanism which cannot be switched off,[3] one can choose to direct or divert one’s attention toward or away from them.

The violin offers several advantages that enable the production of combination tones by a single player alone. The position of the violinist’s ear close to the vibrating instrument’s body makes the perception of combination tones more straightforward. For other instruments which cannot produce two tones simultaneously, two players are required to perform the examples below. In this article – due to the author’s background – the examples have been particularly selected for violinists and other fretless string instrument players, although the phenomenon is no less relevant to wind instruments.

1 The term ‘combination tone’ (*Combinationston*) was introduced by Gerhard Vieth in 1805 (Vieth, “Ueber Combinationstöne,” 265).

2 Sethares, *Tuning*, 83; Taylor, “The Science of Musical Sound,” 58. This is also the author’s experience.

3 Measurements performed on string instruments showed that combination tones can be created also in the instrument itself. However, despite being detected by microphones, they are seldom recognised as such by the ear (Lohri, *Kombinationstöne*, 232–35.).

Combination tone series

When playing a major third with the vibration ratio 4:5, subtle accompanying tones can be heard. These tones do not appear randomly, but according to mathematical law.[4]

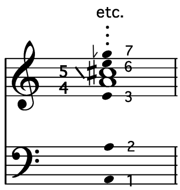

Figure 1, Combination tone series.

Combination tones occur in a harmonic series in which the played interval itself is also embedded. The lowest combination tone, in this article referred to as the fundamental combination tone, is also representative of the mathematical unity of the interval ratio (generator). From a practical point of view, it is noteworthy that this combination tone is the most stable and reliable of the series.[5]

The term ‘difference tone’

Some readers may be familiar with the term ‘difference tone’ and will have been taught to subtract the two numbers of the interval ratio or the two frequencies from each other. This method is problematic because it was not devised for the purpose of musical practice. In his 1856 article “Ueber Combinationstöne”, Hermann von Helmholtz introduced the term ‘difference tone’ and confirmed its calculation *frequency 1 – frequency 2* in the context of experiments with tuning forks which generate ‘simple tones’ (*einfache Töne*).[6] By contrast, for ‘complex tones’ (*zusammengesetzte Töne*) that emanate from musical instruments, he ex-

4 That two tones played simultaneously do not only generate a third tone, but an entire series of tones arranged in harmonic proportions, was recognised by the majority of acousticians by the middle of the 19th century, among them Hällström and Helmholtz (Hällström, “Von den Combinationstönen,” 444 f.; Helmholtz, “Ueber Combinationstöne,” 522).

5 The nature and history of the theory of the fundamental combination tone is of such interest that in the author’s dissertation *Kombinationstöne* an entire chapter is dedicated to the subject. Lohri, *Kombinationstöne*, 175–202.

6 Helmholtz, “Ueber Combinationstöne,” 501 and 518.

plains that the subtraction must been made between all of the possible combinations of overtones.[7]

Surprisingly, fourteen years later in his well-known work *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik,* Helmholtz does not make this distinction clear.[8] Moreover, he uses the musical notation system associated with natural tones (tones rich in overtones), while showing results made by experiments using 'simple tones' (tones poor in overtones). Consequently, musicians, theorists and scientists may be tempted to use an inappropriate method of calculation—or even to confuse it with Giuseppe Tartini's calculation of the *terzo suono*.[9]

In Helmholtz's 1856 article "Ueber Combinationstöne", it appears that also he was aware of the existence of the combination tone series[10] and that he did not intend the difference tone to be used as a working tool for musicians.[11] In general, theories related to difference tone or to nonlinear distortions in the inner ear normally apply to single frequencies (sine tones) and not to musical sounds, which are rich in overtones.[12]

7 Helmholtz "Ueber Combinationstöne," 501, 518 and 522; Helmholtz *Die Lehre von den Tonempfindungen*, 243. When including all overtones, Helmholtz's method of calculation results in a combination tone series of the type *f*, 2*f*, 3*f*, 4*f* etc. Additionally, from a neurophysiological point of view, there would be no explanation as to how the ear should be able to restrict the natural sound processing mechanism to two fundamental frequencies only while suppressing or fading out its partial tones.

8 Helmholtz, *Die Lehre von den Tonempfindungen*, 240.

9 See Lohri, *Kombinationstöne*, 61 ff. Helmholtz apparently studied only Tartini's *Trattato di musica secondo la vera scienza dell'armonia.* He stated that Tartini had written the third tone one octave too high (Helmholtz, "Ueber Combinationstöne," 498). Tartini's later work *De'principj dell'armonia musicale contenuta nel diatonico genere* from 1767 reveals that Tartini changed the formula of the *terzo suono* to one octave below. Indeed, this change happened even earlier, due to the correspondence with Leonhard Euler, around the year 1756 (De Piero, *Il Tentamen*, 20; Lohri, *Kombinationstöne*, 81).

10 Helmholtz, "Ueber Combinationstöne," 528 f.

11 Helmholtz, "Ueber Combinationstöne," 518 and 522.

12 On the other hand, theories which are based on neuronal sound processing and related to the terms 'virtual pitch', 'virtual bass', 'missing fundamental', 'residual tone' or 'periodicity pitch' correlate to a high degree with the observations of combination tones in musical contexts (Lohri, *Kombinationstöne*, 26 ff. and 189 ff.). See also Tadeusz Ziębakowski, "Combination Tones".

Audibility

In practice, the sound signal properties determine to a large extent which of the combination tones take on a perceptible shape, and which maintain an "incognito" or unmanifested state.[13] The more complex the frequency ratio, the more ambiguous and unstable the resulting combination tones. In the sound-processing mechanism of the ear and brain, there is a natural resolving-capacity limit and the neurophysiological apparatus is no longer able to detect the unity or relationship between the tones. Tempered intervals are also subject to this factor and therefore are unable to produce such clear combination tones as just intervals. For tempered intervals "the vital force of the stimulus" is distributed to many "nerve excitements", as Carl Stumpf describes in his *Tonpsychologie*.[14] From this we conclude that combination tones and just intervals exist in a kind of symbiotic relationship.

Benefits for instrumental practice

Quality and quantity

On fretless string instruments, the player has the advantage – and also the challenge – of being free to choose the interval size every time. The question of how to find the appropriate interval is therefore very important.

Regarding combination tones, the minor sixth with the vibration ratio 5:8 is a significant example on the violin (see figure 2, *a'*-*f''*). Here the combination tones that correspond to 1 and 3 are distinctly audible to the author. On the right-hand side of figure 2 is shown the interval 3:5 which corresponds to the major sixth. Here, one can hear not only 1, but also the octave above, 2.[15]

The interval is then realised in its most accurate way when the actual audible combination tones appear at their correct place, matching with the tones of the combination tone series.

13 Tartini, *Trattato di musica*, 17; Helmholtz, "Ueber Combinationstöne," 522.

14 Stumpf, "Beobachtungen über Kombinationstöne," 246.

15 This is not only the author's own hearing perception, but also that of other listeners consulted during research, lessons, presentations or workshops.

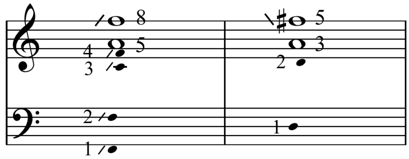

Figure 2, The minor and major sixths and the combination tones that may be perceived below the played notes.

Micro adjustments

No less important for the practice of intonation is the fact that combination tones below the played notes react much more quickly in pitch than the played notes themselves. Combination tones therefore represent a tool for refining the adjustment of the interval. The most sensitive and quick to react is the fundamental combination tone: indeed, this is a true guide for intonation. Furthermore, the more closely one approaches the just interval, the clearer the combination tones become.

By applying this method, the player will increasingly refine their perception and learn to form a clear image of any just musical interval. Careful listening to the sound quality of a particular interval, as well as repeated encounters with the precise ratios, will help to develop the player's ability to play in tune, even in fast tempi.

When the vibration ratio is realised accurately, combination tones act like pilots leading to the exact pitch. At this moment, all perceived tones stand in a harmonic relationship to each other, allowing us to control the smallest shifts in pitch while producing an interval, and to observe and understand their impact on the sound quality. Paul Hindemith alludes to a geometrical exercise:

> They [the combination tones] are the third point of a triangle whose other two points are in the sounding interval, making possible for the ear a sort of trigonometry by which it is enabled to form a judgement of the purity of an interval.[16]

How far one can go with this tool? Is it possible to play whole tones of different sizes on the violin? In other words, is it possible to realise the intervals 9:10, 8:9 and 7:8 solely with the help of the ear? The easiest way to accomplish this task is by choosing an open string for one of the two notes. In this way, the open string becomes a fixed reference tone for the other note whose place must be found by the ear and the finger.

16 Hindemith, *The Craft of Musical Composition*, 58.

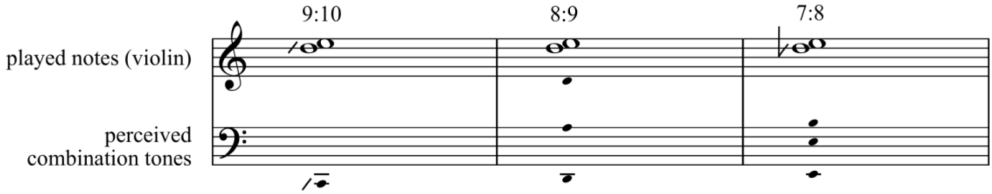

Figure 3, Above, the played notes. Below, the perceived combination tones as performed and perceived by the author. The ascending slash (⁄) stands for the syntonic comma and indicates a note which is higher by 81:80 as compared to a *d* belonging to the chain of fifths. The septimal comma 63:64 is represented by Tartini's arrow (↓) and indicates that the note deviates by 63:64 from the note generated by the chain of fifths.

In order to allow an audience to experience these subtle realms of sound created within the ear or sometimes by the instrument itself,[17] it can be helpful to amplify combination tones with the aid of another instrument. For the violin, a suitable accompaniment is the violoncello or the double bass due to their ability to generate tones in low registers.

The technique shown above allows the identification and realisation of intervals with such precision as to facilitate not only just intonation, but also with the orientation and approximation of any microinterval.

Building syntonic scales

Giuseppe Tartini is often cited as the 'discoverer' of the phenomenon of combination tones. Indeed, he dedicated a large part of his life to the enquiry of what he called the "third tone" (*terzo suono*). Tartini described a combination tone which correlates with the fundamental combination tone or the octave above (only until c. 1756, see footnote 9). He writes:

> When I play in double stops on my violin I can physically meet the form of the interval, whose physical sign and proof is the third tone which must result. I thus have for myself and for my students the benefit of the safe intonation, and therefore of the real use of the aforementioned scale in precision of ratios.[18]

This implies that not only can intervals be measured by means of a reference tone and the resulting *terzo suono*, but entire scales can as well.

17 Lohri, Carral and Chatziioannou, "Combination Tones in Violins".

18 Tartini, *Trattato di musica*, 100. Translation by the author.

In the above citation, Tartini refers to “the syntonic diatonic scale of Ptolemaios”[19]; while Tartini’s example indeed corresponds with the familiar diatonic major scale, the term ‘syntonic’ indicates that every tone of the scale must be in tune with the tonic and therefore must be of integer vibration ratios. The tone material for this scale are the three triads built on *c*, *g* and *f*.

Figure 4, Tartini’s syntonic diatonic scale (the numbers refer to string lengths).[20] Reduced to their lowest terms the intervals are 1:1 (*c*), 8:9 (*d*), 4:5 (*e*), 3:4 (*f*), 2:3 (*g*), 3:5 (*a*), 8:15 (*b*), 1:2 (*c*).

To build this scale on the violin, the following technique can be applied: choose an open string as the tonic and play the notes of the scale on an adjacent string. Playing both strings simultaneously creates a drone scale. Check the pitch with the ‘third tone’ (fundamental combination tone) for each successive interval. If the fundamental combination tone is too low to be heard, check with other audible combination tones of the series. With this procedure it is possible to build the above-mentioned scale in absolutely accurate ratios taking *g*, *d*, *a* or *e* as the tonic on the violin.

Once this exercise has been internalised, the method can be used to build any scale that is composed out of just intervals. The next step would be to expand the tone material to a syntonic chromatic scale. Like the syntonic diatonic scale, the syntonic chromatic scale is also based on the prime numbers 2, 3 and 5.

19 Tartini, *De’principj dell’armonia musicale*, 77. See also Zarlino, *Istitutioni Harmoniche*, 139–40: “Che ’l diatonico syntono di Tolemeo sia quello, che naturalmente ha la sua forma da i Numeri harmonici […] Tuo.mag. – Tuo.min. – Semit.mag. – Tuo.mag. – Tuo.min. – Tuo.mag. – Semit.mag.”

20 Tartini, *Trattato di musica*, 99.

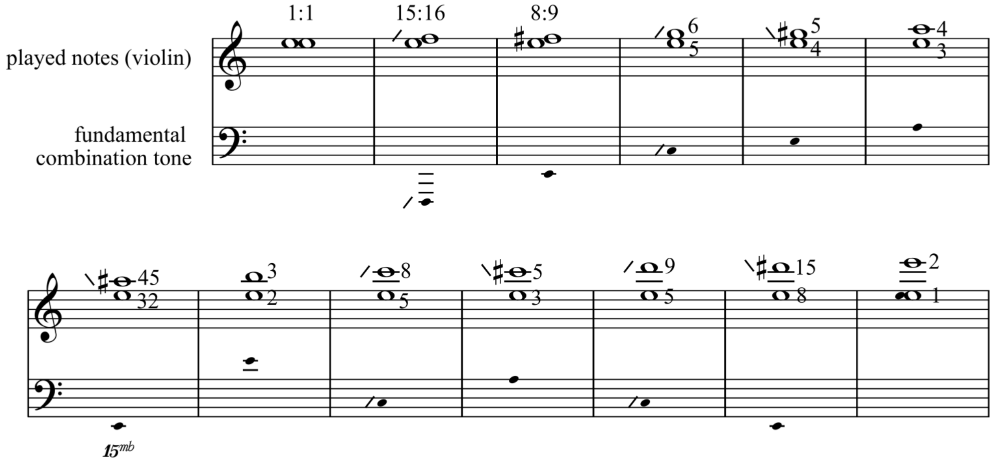

Figure 5, The syntonic chromatic scale played as a drone scale (reference tone *e*).

The relations of tones become clear when depicted on a ‘tone lattice’, an idea originating with Leonhard Euler (*Speculum Musicum*[21]):

\ c# (25)	\ g# (75)	\ d# (225)	\ a# (675)
a (5)	e (15)	b (45)	f# (135)
/ f (1)	/ c (3)	/ g (9)	/ d (27)

Figure 6, Tone lattice with the twelve tones of the syntonic chromatic scale in *e*. The fifths are arranged horizontally and the thirds vertically.[22]

Philippe Borer has investigated this particular scale in relation to Niccolò Paganini. The fact that the great violinist entitled it “Scala di Paganini” suggests that he attached a great importance to these specific tone relations.[23]

By practising these scales regularly, the ear will learn to memorise the form of a just interval, its sound quality on the violin and its resonance in the human body. The player establishes a base for orientation, eventually proceeding to explore more unusual intervals which are based on larger prime numbers like 7 or 11.

21 Euler, *De harmoniae veris principiis per speculum musicum repraesentatis*, 584.

22 “The numbers indicate the rank of each note in the order of their harmonic generation (power of *3* for the fifths and power of *5* for the thirds).” Borer, “The Chromatic Scale in the Compositions of Viotti and Paganini,” 101.

23 Borer, “The Chromatic Scale in the Compositions of Viotti and Paganini,” 97 and 120.

Figure 7, A syntonic scale created by Carl Stumpf in the course of his investigations of combination tones. He uses intervals that create symmetrical combination tone series in mirror image.[24]

In order to approximate tunings that contain tempered intervals such as those frequently found in Baroque and Classical as well as modern music a reliable and objectively accessible method is required. Musicians may feel easily and personally offended when different opinions about intonation clash. For this reason, it is essential to have appropriate tools to work with, such as the control by the *terzo suono*. As Tartini described, it is an objective and "physical sign"[25] of "unity within multiplicity"[26].

Bass effect

The final characteristic of combination tones we shall discuss in this paper is the effect of a bass generated by the *terzo suono* (fundamental combination tone). The fundamental combination tone is a real audible harmonic bass which comes to fruition particularly in solo literature for the violin. Awareness of this fact adds a new dimension to interpretation. Consider J. S. Bach's Sonatas and Partitas for solo violin—a repertoire that makes combination tones very clearly audible to the listeners. In Bach's repertoire for solo violin, intervals that are of equal size on a piano have to be differentiated on a violin. For example, the diminished fifth 5:7 and the augmented forth 7:10 are not the same interval. Using the natural seventh, the ratios 5:7 (diminished fifth) and 7:10 (augmented forth) can be measured by the ear when listening to their specific combination tones (see Fig. 8).

24 Stumpf, "Beobachtungen über Kombinationstöne", 41.

25 Tartini, *Trattato di musica*, 100.

26 Tartini, *Trattato di musica*, 12 f.; "[...] quanto si è rilevato dai soli tre suoni, o dai più, appartiene alla superficie, e non al centro della sua intrinseca natura." (Tartini, *De'principj dell'armonia musicale*, 5); Lohri, "Combination Tones – Unity and Multiplicity."

Although the alternation between 5:7 and 7:10 can require the player to shift only the weight of the finger slightly without even lifting it, the intervals generate a very different bass tone (combination tone) and therefore also demand a different harmonic resolution.

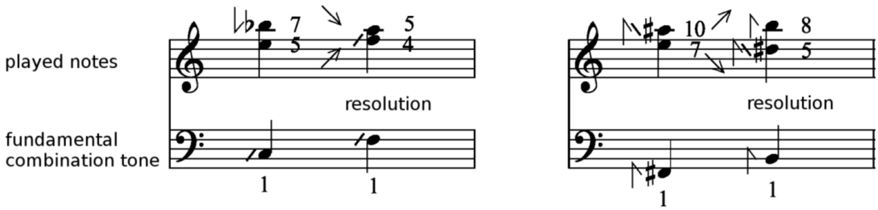

Figure 8, On string instruments there is a perceptible difference between the stopping location of *b-flat* and *a-sharp*.[27] Here are shown 5:7 and 7:10 in their accurate notation. For comparison, both intervals are built upon the reference tone *e* (open string). The diagonal ascending or descending strokes stand for the syntonic comma (80:81, respectively 81:80). The comma 63:64, respectively 64:63 is represented by 'Tartini's arrow' and indicates that the note deviates by 63:64 from the note generated by the chain of fifths. The oblique arrows show the direction of resolution.

In the above example, the difference between *b-flat* and *a-sharp* is apparent in the notation. Some cases, however, particularly in solo violin repertoire, present greater challenges in the form of ambiguities that ask the performer to make a conscious decision on how to play what is notated.

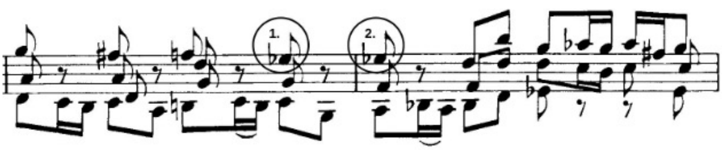

Figure 9, Excerpt of J. S. Bach's Fuga for Solo Violin, Sonata I, BWV 1001, measures 83 and 84). In order to produce a harmonic bass accompaniment – in form of combination tones –, the first *e-flat* should be played perceptibly higher than the second one. The first *e-flat* relates to *c* as 5:6 (minor third) whereas the second *e-flat* is the natural seventh of *f* (4:7). For the first *e-flat* the sign of the ascending syntonic comma could be used (deviation of 81:80 from the chain of fifths), for the second *e-flat* the descending arrow of Tartini (deviation of 63:64 from the chain of fifths).

This applies to ensemble music as well. Here is the opening of Antonio Vivaldi's *Autunno* in two options of intonation:

27 Note that here, because of its accidentals, *b-flat* is lower than *a-sharp*. Thus, the augmented forth 7:10 is larger than the diminished fifth 5:7.

Figure 10, Beginning of Antonio Vivaldi's *Autumn* in two performing options. On the upper stave the played notes, on the bass stave the resulting fundamental combination tones (*terzi suoni*).

If the third *g″–b flat″* is played as 5:6, the composer's bassline is not respected: the fundamental combination tone *e-flat* appears instead of Vivaldi's *c*. If the third *g″-b flat″* is played as 6:7 however, the resultant tone is *c*.

Thus, the performer is presented with a certain scope within which to create an implied bass line at will—a freedom that exists because our standard, limited notational system does not always specify the ratio of a given interval. When embedded in different harmonic contexts, the apparently identical third can indeed take on two distinct intervallic identities: either 5:6 (minor third) or 6:7 (septimal third).

The general use of equal temperament and the modern piano has somewhat disguised the real idea that stands behind each interval. On the piano, the distinction between *f-sharp* sharp or *g-flat*, for example, has become a functional question, whereas on a fretless instrument, the flat and sharp symbols indicate different tones, and therefore a different interval which generates its own specific bass tone when played accurately. An awareness of combination tones helps to reveal the nature and origin of the interval in question and thus improves the general understanding of tunings and tone systems.

Conclusion

The suggestions, exercises and insights found in this article may inspire the reader to deepen their knowledge of intonation. Once the awareness of just intervals and syntonic scales is developed in theory and practice, a string player will find it easier to approach tempered scales, such as mean-tone or equal temperament. It is the author's hope that these findings may become increasingly integrated into the teachings of music schools and academies, and that the research will continue to the degree that intonation becomes recognised as an essential field of sound exploration and to the development of differentiated hearing. This, in turn, could stimulate instrumentalists to broaden their horizons and to explore Eastern music traditions that require also the use of a great palette of intervals.

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