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Music Theory: Speculation, Reasoning, Experience

A Perspective from Systematic Musicology

Albrecht Schneider

Music theory at present comprises a broad range of concepts and approaches. Besides historical research devoted to the reconstruction and interpretation of source material relevant to ('western') music theory, one finds studies with a focus on the description of individual works or on the 'poetics' of certain composers. In this respect, music theory is closely connected to music analysis and aesthetics. In the past decades, perceptual and cognitive issues have been included more often also in studies in music theory and analysis. Further, music theory is regarded to make up an 'applied' discipline whose goal is to teach basic musical skills, and thus to serve a 'practical' purpose, in the first place. Though these orientations all seem legitimate and of relevance to music theory, one should neither neglect the philosophical and scientific background of the discipline nor the need to carry out empirical and experimental investigations that have been part of music theory since antiquity. In the present article a perspective mainly chosen from the history of ideas is combined with some empirical data and demonstrations in order to elucidate a concept of music theory that continues traditions of fundamental music research.

Musiktheorie umfasst gegenwärtig eine Vielzahl von Ansätzen und Ausrichtungen. Auf der einen Seite steht die Geschichte der (abendländischen) Musiktheorie und des musikalischen Denkens in verschiedenen Epochen, auf der anderen finden sich eher handwerkliche und musikalisch-didaktisch ›angewandte‹ Konzepte, die vielfach auf den Unterricht an Schulen und Hochschulen abzielen. Hinzu kommen zahlreiche Studien, die zwar mit Musiktheorie zu tun haben, im Kern aber vor allem Techniken der musikalischen Analyse an einzelnen Werken erproben oder die musikalische ›Poetik‹ einzelner Komponisten erläutern wollen. Die auf diese Weise oft gegebene Verbindung von Musiktheorie zur musikalischen Analyse und auch zur Musikästhetik ist in der Sache begründet und also legitim (gleiches gilt für den ›angewandten‹ Bereich). Die heute oft teils musikhistorisch und werkästhetisch, teils praktisch gelagerten Ausprägungen von Musiktheorie dürfen aber die philosophischen und naturwissenschaftlichen Grundlagen der Musiktheorie ebenso wenig in Vergessenheit geraten lassen wie die Notwendigkeit einer empirischen Untersuchung von Sachverhalten aus dem Bereich der musikalischen Wahrnehmung und Kognition. Musiktheorie zielt nach diesem Verständnis auf musikbezogene Grundlagenforschung.

1. Introduction

The term 'music theory' carries more than one meaning. It may denote, on the one hand, an approach in which theory concerning music in some sense is fundamental, as in works devoted to the very 'logic' of music or to its equivalent, a thorough mathematical

treatment of musical elements and structures.¹ Also, one may think of a philosophical or anthropological theory of music that would qualify as ‘music theory.’² On the other hand, many works usually reckoned among ‘music theory’ serve a practical purpose, in the first place. In this respect, one may point to the multitude of textbooks on harmony, many of which were written by composers like Nikolai Rimsky-Korsakov, Arnold Schoenberg, or Walter Piston. Of course, such works may contain some genuine ‘theory’ (meaning rational constructs based on certain axioms or similar fundamental assumptions as well as including hypotheses, models and a coherent treatment of certain problems) besides a more practical orientation. For example, Paul Hindemith’s *Unterweisung im Tonsatz* contains a proposal of how to calculate a musical scale from harmonic partials,³ and draws substantially on the psychoacoustic phenomenon of combination tones. Schoenberg’s *Harmonielehre* (1911), on the other hand, in its final chapter offers some ideas concerning the possibility to create a ‘Klangfarbenmelodik.’ The remarks he gives are not quite clear, and hypothetical at best; one must, therefore, try to ‘translate’ his ideas into an empirically meaningful description suited to allow experimental investigation.⁴

Currently, music theory may include subjects ranging from elementary harmony and counterpoint to advanced models of musical grammar and syntax, and may be based on various methods ranging from solfège and demonstrations at the piano to computer simulation of generative processes (e. g., improvisation). There are some comprehensive historic surveys of approaches to ‘western’ music theory (from antiquity to the first half of the 20th century⁵); also, some major publications focus on more recent research trends, many of which have been heavily influenced by the cognitive sciences.⁶

The complexity of matters of course increases if one attempts to cover music theoretical systems of areas such as Northern and Southern India, Persia, the Arab cultures or major parts of China. One simply has to notice that research on non-western music cultures and their theoretical systems including conceptualizations (either accessible from written sources or, if transmitted only in an oral format, by fieldwork) has been intensified over the past decades.⁷ Of course, there are concepts of ‘music theory’ found in particular in Near and Middle Eastern cultures (as well as in parts of Northern India) that not only converge to some extent among each other but which are historically related also to ‘classical’ Greek music theory.⁸

1 For example, Harburger 1919, Tanguiane 1993, Noll 1995, Marsden 2000, Mazzola 1985, 1990.

2 For example, Nachtsheim 1981, Karbusicky 1990.

3 Hindemith 1940. As Handschin (1948, 130f.) in his critique of Hindemith has argued, the approach chosen by Hindemith in fact was first outlined by d’Alembert in the (revised) 1762 edition of the *Éléments*.

4 See Schneider 2000.

5 See the volume edited by Christensen (2002). The German project *Geschichte der Musiktheorie* (Zaminer 1984 ff.) is intended to replace Riemann’s classic work (1898) by a new edition based on a critical re-examination of sources and on relevant research done in the 20th century.

6 See, for example, Krumhansl 1990, Seifert 1993, Leman 1995, Huovinen 2002.

7 Much of the relevant research has been summarized in *The Garland Encyclopedia of World Music* (10 volumes). New York: Garland Publishing 1998–2002.

8 See, for examples, articles in Elsner 1989, 1992; Zannos 1994.

Though music theory is by no means restricted to ‘western’ culture, it gained a special status first in Greek antiquity,⁹ and later in European as well as in Asian countries that participated in the lore that had been established in Greece and in Greek colonies in the south of the Italian peninsula.¹⁰ In the section following this introduction, I will briefly relate some aspects of the notion of ‘theory’ as outlined in Greek and medieval sources in order to elucidate the role theory had in regard to fundamental research done in music and mathematics. According to Aristotle,¹¹ ‘theoria’ means a certain mode of mental activity pursued by subjects who conduct reasoning for its own sake (with little regard to ‘practical applicability’).¹² The term ‘speculatio’ that is found in many Latin sources in certain respects was closely related to the Greek ‘theoria’ (the Greek term ‘theoria’ often was translated into Latin as ‘speculatio’). Terms like ‘speculator,’ meaning ‘scout,’ referred to a certain activity, that of investigation (there was also the term ‘speculator naturae’). It was in the context of Christian theology and philosophy that ‘speculatio’ gained a somewhat different meaning, including aspects of contemplation and meditation. Though ‘theoria’ and ‘speculatio’ allow for contemplation, it does by no means preclude scientific rigor. In a treatise on music-mathematics usually believed to be quite speculative, Kepler’s *Harmonice Mundi* (1619), there is a thorough mathematical background.¹³ Moreover, ‘speculation’ as a means for ‘hypothetical mental construction’ can be regarded as part of scientific heuristics.

As far as Greek music-mathematics and science in general are concerned, there are many indications showing that ‘theoria’ was accompanied by some form of experiment, or that theoretical concepts emerged from insight first gained by experience.¹⁴ The combination of theoretical analysis and empirical research became characteristic for the sciences since the time of Aristotle (and probably even earlier). Systematic musicology that in fact can trace its beginnings to scholars like Archytas, Euclid, and Ptolemy (Klaudios Ptolemaios) pursues such a perspective to the present, though of course with different objectives and methodology.¹⁵ However, the impact Greek music-mathematics and music theory had on research still in the past century is obvious from scholarly studies and monographs such as published by Carl Stumpf (1897, 1898, 1901), Jacques Handschin (1948), and Martin Vogel (1975, 1993).

This fact alone contradicts a claim made by Dahlhaus¹⁶ who held that Greek concepts of music theory, and in particular the Aristotelian notion of ‘theoria’ has been “completely” (*restlos*) eliminated in modern times in Europe. The new paradigm for mu-

9 See Barker 1989, 2007. Not to complicate matters, I leave out the issue of origins parts of Greek music theory may have had in the Orient (for example Babylon) and in Egypt.

10 Kittler (2006, chapter 2) provides insights in the cultural context of Greek music-mathematics (as developed, in the main, in the cities of Magna Graecia) from Pythagoras to Archytas.

11 Arist. pol. 1325b.

12 See also Düring 1966, 468–73.

13 See Stephenson 1994, and below in this text.

14 See Szabó 1978, Barker 1989, and below in this text.

15 See contributions in Leman (ed.) 1997, Schneider (ed.) 2008b, Bader (ed.) 2009.

16 Dahlhaus 1985, 14.

sic theory, as Dahlhaus¹⁷ saw it, ultimately became (historical) understanding of musical works in regard to their conception and reception. This view of course contains some plain truth: one of the tasks of music theory is to study works of music in regard to structural elements and features, harmonic textures, etc. However, in order to match the notion of ‘theory,’ the study of individual works of music will go beyond technical analysis and description, and may focus on, for example, semiological aspects,¹⁸ or specifics of sound structure as dependent on instrumentation and playing techniques.¹⁹ Also, an analysis (even of single works) not confined to a technical description and hermeneutic interpretation may shed light on more general concepts relevant to sound research, music perception and cognition.²⁰

Music theory is by no means restricted to the analysis of single works, or to the description of individual composer’s ‘poetics.’ Such a view would unduly narrow the scope of the discipline. One will notice that Schoenberg, in his *Harmonielehre* (which in part serves the cause of justifying Schoenberg’s own approach to composition around 1910), maintains a view that is fairly broad in regard to techniques explored in various styles of tonal music.²¹ Similarly, Hindemith’s *Unterweisung* (1940) indeed attempts to present fundamentals of tonal music (with examples spanning various styles and eras) in a textbook format. In addition, there are comprehensive works on tonal music that offer a descriptive and analytic approach including aspects of perception and apprehension.²²

Further, music theory has been pursued from a more formal perspective. Thereby, musical elements, structures, and also concepts as such have been subjected to a logical or mathematical treatment.²³ Other approaches are based on concepts derived from structural linguistics (best known perhaps is Lerdahl and Jackendoff 1983). What is common to many contemporary studies in music theory is an orientation towards cognitive aspects, and a methodology that includes algorithmic modelling, signal processing, and/or behavioural experiments.²⁴ In sum, there is a huge amount of ‘theory’ (conceptualization, system design, etc.), on the one hand, and a tendency to back theoretical reasoning by empiricism, on the other.

2. Theory, Speculation, Empiricism

The beginnings of music theory in Europe, usually attributed to Pythagoras and his school, reveal the importance of numbers, ratios and proportions employed to determine relations of tones and their possible use as intervals. Of course, the ‘logos’ inherent in

17 Dahlhaus 1985, 15.

18 See, for example, Nattiez 1982.

19 For example, Beurmann and Schneider 1991, Petersen and Schneider 2003, Reuter 2002.

20 For case studies, see Schneider 1995, 2000, 2005; Bader 2005.

21 See Rexroth 1971.

22 For example, Erpf 1967, Albersheim 1980, Kresánek 1994.

23 For example, Wille 1976, Mazzola 1990, Noll 1995. See also Seifert 1993.

24 See Leman 1995, Lerdahl 2001, Huovinen 2002, Frieler 2009.

harmonic relations ('logos' meaning relation between two numbers²⁵) and their grouping into proportions (representing ratios of ratios, for example $6:9 = 8:12$ and $12:9 = 8:6$) is of interest as such, on the one hand, and may be applied to fields other than music, on the other. When Aristotle, in his *Metaphysics*, criticized the "... so-called Pythagoreans ..."²⁶ he did so because they relied, in his view, too much on numbers as well as on relations (as expressed by numbers). Aristotle apparently did object to treating different spheres (e.g., arithmetic, acoustics, music) along the same basic theory. In modern science, mathematical principles and techniques indeed serve as the unifying factor, and mathematics is still most useful for the analysis and description of musical and acoustical phenomena.²⁷

Mathematics of the Pythagoreans reconstructed from Greek sources²⁸ revealed that basic musical intervals (octave 1:2, fifth 2:3, fourth 3:4, major second or 'tone' 8:9) could be derived not only by proportions (the Tetraktys 6:8:9:12)²⁹ built from natural numbers (1, 2, 3, 4) and their multiples but also by geometrical operations. Thereby, one could find additional intervals; for example, the tone 8:9, taken as a segment that is subtracted twice from the section [of a string] representing a fourth 3:4, yields both the 'Pythagorean' major third 64:81, and, as a remainder, the 'leimma' of 243:256, which is usable as a melodic step. Once it was understood that a line AB of given length can be divided into sections according to certain principles (e.g., the arithmetic, the geometric, and the harmonic mean, respectively³⁰), one could apply these principles to a string of a musical instrument or to any other one-dimensional continuum where fixed points constitute intervals among each other. As Ptolemy states, Archytas of Tarent understood how to divide a fourth (4/3) into a Tetrachord whose intervals either conform to the enharmonic, to the chromatic, or to the diatonic genus:³¹

$$5/4 \times 36/35 \times 28/27 = 4/3; 32/27 \times 243/224 \times 28/27 = 4/3; 9/8 \times 8/7 \times 28/27 = 4/3.$$

These ratios include the pure major third 5/4, the 'Pythagorean' minor third 32/27, the major whole tone 9/8, and a large whole tone 8/7 (\approx 231 cent). This interval as well as that of 7/6 (\approx 267 cent) is found in descriptions of Tetrachords, of which Ptolemy says they were in use among the players of the kithara.³² Given the scheme of constructing intervals by sequences of pure fifths,³³ one would expect a major third defined as a Ditonos (81/64) instead of the pure major third 5/4. However, it takes but little effort to change the procedure so that ratios of 5/4 and 6/5, or, alternatively 7/6 and 8/7 will be

25 See Szabo 1978, 168ff.

26 Arist. met. 985b.

27 See Neumaier 1986, Schneider 1997a, Fauvel et al. 2003, Benson 2007, Mazzola et al. 2002.

28 See van der Waerden 1947/49, Szabó 1978.

29 See Vogel 1993, Kittler 2006.

30 See Szabó 1978, part 2, Vogel 1993, 31–35; Barker 1989, 42, 47ff.; 2007, 302f.

31 Ptol. harm. p. 30, 31, ed. Düring 1930.

32 Ptol. Harm. p. 42, 43 ed. Düring 1930. The ratio 8/7 is also found in Al-Farabi (Manik 1969, 14f.), and in recent works on Greek and Byzantine music theory (see Zannos 1994, 96).

33 As explained by Plato Tim. 35a–36d; see Handschin 1948, 360ff.

obtained.³⁴ The *Sectio canonis* attributed to Euclid demonstrates how divisions and multiples of sections (of a single string) can be used to establish the fundamental structure of the Greek tone system.³⁵

It is quite likely that calculations accompanied experimental work. As Arpád Szabó states, basic concepts such as ‘diastema’ (= interval) “... could never have been originated without musical experiments using the canon.”³⁶ It is likely that Archytas “... attempted both to analyse the attunements underlying contemporary musical practice, and to reveal the principles of mathematical order on which they were based.”³⁷ Even seemingly abstract constructs concerning numbers and magnitudes, which Aristotle³⁸ attributes to Plato and (before him) the Pythagoreans, and finds cumbersome to explain, can be translated into a scheme that includes all basic music intervals.³⁹ Further, one will note that the two series disclosed in Plato’s *Timaios* (35b), which can be written as 1 : 2 : 4 : 8 (= 1 : 2¹ : 2² : 2³) and 1 : 3 : 9 : 27 (= 1 : 3¹ : 3² : 3³), were discussed also in much later writings on music theory (with more ‘practical’ objectives, as in Rameau 1750; see below). In general, Greek mathematics can be translated into modern formulations. One may point to the so-called Aristoxenus/Euclid-controversy concerning the division of the whole tone 8/9 into two or more (equal) parts. The proof Euclid laid down to show that it is not possible (with respect to a geometric series) to insert a natural number between two natural numbers whose relation is (n+1) : n can be stated in terms of modern arithmetic.⁴⁰ Similarly, the problem Ptolemy raises⁴¹ whether the Aristoxenians were correct saying that the octave comprises six whole tones (9/8) can be easily subjected to algorithmic probing. Whereas Ptolemy had to rely on fractions to find the difference between (9/8)⁶ = 531441/262144 and the octave (2/1), which is 7153/262144, one can now proceed by finding a set of solutions which must satisfy the equation⁴²

$$\begin{aligned} &\text{Solve}[(x^6) / (8^6) = 2, x]//N \text{ and, alternatively} \\ &\text{Solve}[(9^6) / (x^6) = 2, x]//N \end{aligned}$$

The set of solutions includes $x = 8.9797$ for the numerator, and $x = 8.01809$ for the denominator. Hence, the problem of giving six whole tones such a size as to make up exactly one octave, would be solved if the ratio of fundamental frequencies between each two tones would equal a real number $k = 1.12246$, which in turn corresponds to 200 cents (the interval of the whole tone in 12-tone equal temperament). Such a calculation (which

34 See Barker 1989, 48f.

35 See the German translation of Euclid’s ‘Katatomé Kanonos’ (sectio canonis) in Busch 1998, 133 ff. (including graphics). It is of no particular interest here whether or not Euclid is the author of the work in question.

36 Szabó 1978, 121.

37 Barker 1989, 51.

38 Arist. met. 987b20ff.

39 See Becker 1957, 6 (and the review by Kurt von Fritz in *Gnomon* 33, 1963, 7).

40 Neumaier 1986, 127f.

41 Ptol. Harm. p. 59 ed. Düring 1930.

42 Written in the syntax of Mathematica.

involves logarithms) was neither available to Aristoxenus nor to Ptolemy. The ‘overshoot’ of the six whole tones ($9/8^6$) over the octave ($2/1$) of course sums up to 24 cent (a ‘Pythagorean’ comma). Ptolemy objected that neither calculation (based on fractions) nor tuning an eight-string ‘Kanon’ and listening to the resulting pitches would confirm Aristoxenian opinions. His own estimate of the difference between six tones ($9/8^6$) and the octave is about $65/64$ (≈ 27 cent), which is close to the exact numerical solution, and within the limits of auditory discrimination.

Notwithstanding possible ‘practical’ applications,⁴³ Pythagorean music-mathematics was a field of ‘theoria’ in the sense that the logical coherence of arguments and explanations is the decisive criterion which philosophical, mathematical or any other scientific investigations must fulfil. According to Aristotle,⁴⁴ in particular sciences dealing with causes and principles (e.g., physics, philosophy) are ‘theoretical’ in that their approach is analytic and reflective rather than suited to guide human behaviour or (artistic or other) production. When it comes to scientific reasoning, there is a clear hierarchy for Aristotle leading from perception to experience and then further on to true knowledge.⁴⁵ The analytic (and in part contemplative) nature of ‘theoria’ as pronounced by Aristotle as well as Platonic ‘ideas’ were well preserved in the long tradition that can be traced from Greek and Hellenic sources to Boethius and the Middle Ages. It included basic essentials of Pythagorean lore,⁴⁶ which was still of influence in the 16th and 17th centuries, respectively. However, by that time a basically ‘Pythagorean’ way of thinking had been complemented more and more by empirical observation (as is obvious in writings of, for example, Zarlino and Kepler, respectively; see below).

As to science in regard to nature, Aristotle combines analytic inquiry with consideration of factual evidence.⁴⁷ One would not call him, though, an ‘empiricist’ (which has modern connotations). His one-time student, Aristoxenus of Tarent, has been lauded for having introduced ‘strict empiricism’ into the study of music. No doubt there are significant differences between scholars like Archytas and Euclid (who proceed from a mathematical background to the study of empirical phenomena), on the one hand, and Aristoxenus (who had a thorough musical understanding and first-hand experience of musical practice), on the other. Of course, there were controversial arguments concerning, for example, the nature and size of intervals (viewed either as a mere ‘distance’ between two points, or as a relation of two tones resulting in a certain musical and perceptual quality) as well as other issues.⁴⁸ However, there is some common ground shared by ‘Pythagoreans’ and ‘Aristoxenians,’ and there have been attempts to present a

43 One field where Pythagorean proportions (including 3:4:5) certainly played a role was architecture. For recent findings and empirical data, see Birnbaum 2006.

44 Arist. met. 1064a10ff.

45 Aristotle met. 981b–982a. The Aristotelian terms would be *aisth sis*, *empeiria*, and *epist me*, respectively. As to the philosophical background, see for example Mittelstra  1974, ch. 2, and Falcon 2005.

46 See M nchelhaus 1976.

47 See Falcon 2005.

48 For details, including the ‘Aristoxenos/Euclid-controversy,’ see Neumaier 1986, Busch 1998, Barker 1989, 2007.

more integrative approach, in which mathematics, acoustics, and perception are taken into account as is the case in the *Harmonics* of Ptolemy. He clearly saw the bias in the work of some of his predecessors, on the one hand, but could make use of all the previous findings, on the other.

There is no need to emphasize that a science of music (*musica scientia*) needs both orientations, analytic 'theoria' as well as experimental work. The Greek 'Kanon' was in fact a device to carry out measurements, and the monochord in medieval times was reckoned as an instrument that could and should be used for measurement and 'probing'.⁴⁹ At the same time, it was regarded fundamental to demonstrate the correctness of calculations. The monochord, in this respect, is a geometrical tool as well as a 'principle.' The multitude of proportions (many of them representing Greek tone systems and music theory) found in sources like the *Speculum musicae* (ca. 1330) of Jacobus of Liège reflects the erudition of the time.⁵⁰ However, there were factors that brought about change in regard to music theory as well as to science in general. Scholars certainly were taking up and expanding Aristotelian 'empeiria' (as was advocated by Roger Bacon⁵¹). There are indications that, probably beginning around 1300, empirical investigations gained more of an interest. One may point to early contributions to the theory of vibration (including elements of pitch perception) made by Pietro d'Abano of Padua around 1310,⁵² and to Nicole Oresme (1320–1382) who it seems was discovering harmonic partials.⁵³ Also, one has to remember that building musical instruments such as organs required both calculations and some experience with respect to the mensuration of pipes. Apparently during the 14th century, a strictly Pythagorean approach to mensuration based on simple integer proportions (that seems to have governed the type of medieval organ known as 'Blockwerk'⁵⁴) was followed by variable mensuration that lend itself to experimentation. Constructing organs with several ranks of pipes that had different length (usually expressed in foot, e. g. 8', 4', 2 2/3', etc.) meant that pipes were sounding harmonic intervals simultaneously even if one would have played a single melodic line.

Probably the oldest preserved organ in Europe still playable is that of Sion (Valais; Sitten, Wallis). Three stops in 4', 2' and 1 1/3' foot that can be dated to ca. 1400 are extant.⁵⁵ As regards the music that might have been played on an organ corresponding to the Sion instrument in the 2nd half of the 14th century, the repertoire would have included instrumental adaptations of various vocal forms as well as maybe some original pieces

49 Besides sources discussed in Münxelhaus (1976) one may point to Odorannus de Sens (ca. 985–ca. 1045) who gives a lucid instruction of how to build and to use the monochord in order to explore tone systems. See Schneider 1984, 288 ff.

50 Jacobus Leodiensis *Speculum Musicae*, Lib. I–VII, ed. R. Bragard. See in particular Lib. V.

51 See the article on Roger Bacon in the *Stanford Encyclopedia of Philosophy* (<http://plato.stanford.edu/entries/roger-bacon/>), and in particular the paragraph on Bacon's *De scientia experimentalis* (1266–67).

52 See Palisca 2006, 135 ff.

53 See Taschow 1997.

54 See Klotz 1975, Sachs 1980.

55 See Klotz 1975 and the recordings plus commentary available on the LP *Historische Orgeln der Schweiz*, E. Power Biggs (CBS S 72441, n. d.).

conceived for execution on the organ.⁵⁶ Most of the musical settings show parallel intervals (with a predominance of the perfect fifth, and a rare major third occurring here and there) so that listeners were exposed to consonant intervals enriched by the harmonics of the pipes used for the respective piece. The beginning of the Estampie *Retrove* from the Robertsbridge codex, when played on the Sion organ, yields a spectral energy distribution as shown (for a frequency range from 0.2–3 kHz) in figure 1. It is obvious that the music, if considered in terms of melodic and ‘vertical’ interval proportions actually put into sound, gains a harmonic dimension even when performed on a small ‘Gothic’ organ. Though the interval proportions were regarded as constitutive elements of the syntactic structure, and may be understood as such by appreciative listeners, the experience of music as performed must have added to the insight that the concept of ‘consonance’ in fact relates to sounds that ‘fit’ together.

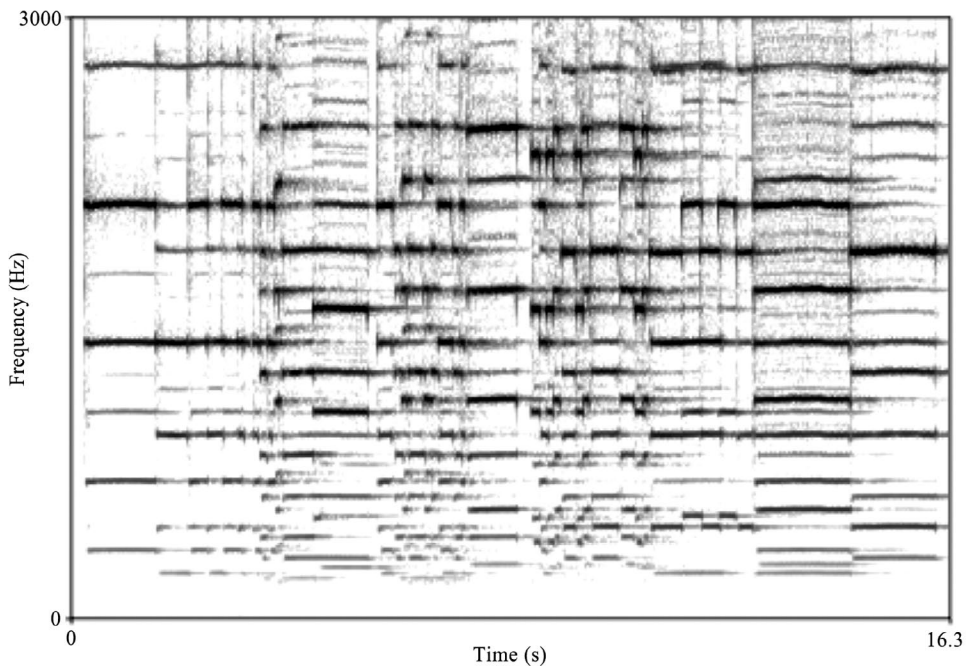


Figure 1. Spectrogram (0.2–3 kHz) of the first phrases of the Estampie *Retrove*, played on the organ of Sion (Switzerland)

One can see that evolving musical practice as well as a different evaluation of actual sensation and perception of music were factors that, by about 1400–1500, began to challenge traditional views.⁵⁷ Though there was still a fair amount of speculative ‘theoria’

56 One might think of organa and conductūs, on the one hand, and of the instrumental pieces (Estampie) included in the Robertsbridge Codex (ca. 1350), on the other.

57 See Palisca 2006, chapters 3 and 8.

found in treatises on music theory, one has to take into account significant developments in musical acoustics and psychoacoustics that were achieved from the 16th century through the 17th and 18th centuries, respectively.⁵⁸ Areas of research and experimentation that became important included (a) the theory of vibration, the phenomena of resonance and harmonic partials, (b) pitch, pitch perception and pitch relations, (c) tuning and temperament, (d) affective and gestural (rhetorical) quality of music. These fields were part also of music theory, and connected in various ways to musical practice.

For example, Zarlino, who still preferred numbers and proportions as principles for rational deduction of musical scales and intervals, offers a discourse on how notes either high or low in pitch apparently relate to the speed of vibration of a string as well as to its diameter and the tension with which it is stressed.⁵⁹ The explanation he offers in part is correct; however, there are some flaws since he seems to believe that the frequency of vibration of a string of given length, tension and diameter would depend, at least to some degree, on the amplitude of vibration.⁶⁰ As the amplitude decreases continuously after a string has been plucked, this would result in decreasing frequency of vibration, and hence in a pitch glide downwards. Beeckman, Descartes, Galilei and others noted that this was actually not the case. The correct solution, which needs a theory of isochronous periods of vibration, was found only in the course of the 17th and 18th centuries, respectively.⁶¹

Kepler, after discussing the minor and the major genus and divisions of the octave according to both,⁶² finds an optimum solution for a just scale per octave, which, if we transform his list of numbers {2160, 2048, 1920, ... , 1080} into common interval ratios plus modern cents,⁶³ reads as stated in Table 1.

Taking this scale resulting from both axiomatic theory and principles of 'natural' hearing as a standard, Kepler compares it to another solution that Vincenzo Galilei (1581) had proposed for a scale to be tuned on the lute. Kepler admits that Galilei, by applying 'mechanical' principles, namely a sequence of divisions based on the ratio 18/17 (yielding intervals that are nearly 99 cents, and, hence, very close to equal temperament), had found a scale tolerable to the ear (as it would allow certain deviations from just interval

58 For in-depth studies, see Truesdell 1960, Cannon and Dostrovsky 1981, Dostrovsky and Cannon 1987, Cohen 1984, Crombie 1990.

59 Zarlino 1558, parte II, cap. 11.

60 The view Zarlino (1558, 78) outlines is that strings for high notes undergo fast movements, and strings for low notes slow movements (equal plucking force assumed). This observation is correct and implies the notion of 'frequency' of vibration. Zarlino also correctly notes that the sound of strings for high notes lasts much shorter than the sound of strings tuned to low notes. He infers from this that the speed of vibration of the string, and the interaction between string and surrounding air, determines the duration of the sound emitted from the respective string (this aspect relates, implicitly, to radiation and consumption of energy as well as to viscous damping).

61 See Truesdell 1960, Dostrovsky and Cannon 1987.

62 Kepler 1619, Lib. III, c. 8.

63 Intervals and cents are also given as cumulated. There are very small errors in the cent values resulting from interpolation that, in sum, amount to 0.6 cent at the octave. The scheme of interval ratios corresponds to the reconstruction of the chromatic scale as discussed in Ramos de Pareja's *Musica Practica* (1482, see Fend 1989, 403).

Note	Interval	Interval cum.	Cent	Cent cum.
G		1/1		0
	135/128		92.2	
G [#]		135/128		92.2
	16/15		111.8	
A		9/8		204
	16/15		111.8	
B ^b		6/5		315.8
	25/24		70.7	
B		5/4		386.5
	16/15		111.8	
c		4/3		498.3
	135/128		92.2	
c [#]		45/32		590.5
	16/15		111.8	
d		3/2		702.3
	16/15		111.8	
e ^b		8/5		814.1
	25/24		70.7	
e		5/3		884.8
	16/15		111.8	
f		16/9		996.6
	135/128		92.2	
f [#]		15/8		1088.8
	16/15		111.8	
g		2/1		1200.6

Table 1. Kepler's calculation of a just scale G–g

proportions) yet not satisfying in the end because the ear, guided by the mind, would not take a ratio of 100.000:50.363 as representing a perfectly consonant interval. Hence, the ear is viewed as a factor relevant to finding practical solutions that, however, may not be acceptable to the musical mind seeking rational analysis. To Kepler, the ear alone cannot provide the degree of exactitude required for confirming models of musical scales and intervals, which thus is left to mathematical justification.

From what has been outlined so far, it should be clear that music theory was closely related to the Aristotelian notion of 'theoria,' and to a general concept of science that combines rational analysis with empirical investigations. Music theory, in this respect, was part of fundamental research (*Grundlagenforschung*), and also intrinsically tied to philosophy.

The argument that has been put forward repeatedly now is that already the 'musica poetica' of the 16th and 17th centuries led to a paradigm shift, whereby subjects traditionally covered under the heading of 'musica theoretica' are said to have lost as much in interest as subjects bearing on techniques of composing actual music did win. Though one may point to various (if not many) writings treating aspects of musical syntax, musi-

cal rhetoric, etc.,⁶⁴ one should not forget that, first of all, fundamental research in acoustics, organology, sensation and perception of musical sound developed rapidly during the same era⁶⁵ so that ‘musica theoretica’ flourished again (though taking some new directions close to mathematics, physics, medicine, etc.). Given the factual evidence, it would be inappropriate to say that ‘musica poetica’ (a movement that rose to prominence especially in parts of Germany) did supersede ‘musica theoretica’ (as continued and developed by Zarlino, Vicentino, Galilei, Descartes, Mersenne, Doni, Kepler, Sauveur, Huygens, Euler, Tartini, etc.). Moreover, a significant portion of ‘New Music’ composed around 1600 did profit not only from antique music theory (even if misinterpreted in certain ways), but also from the experimentalism that had become a true paradigm.⁶⁶

In the following sections, I will further explore relations between theoretical reasoning, speculation (touching on both heuristics and imagination), and empiricism in regard to music theory. Accepting the limitations of an article, a selection of case studies will be presented, each of which seems suited to discuss issues that have been of relevance to music theory. The first case study considers relations between harmonically differentiated compositional structure (as found in chromatic and enharmonic settings from the 16th century onwards), tunings, and intonation. Next, Rameau’s interest in acoustical phenomena (often disapproved of as a shift towards ‘physicalism’ in music theory) is given a brief assessment; there are some recent attempts at giving harmony (psycho)acoustical foundations, which relate to Rameau’s ideas one way or another. Since Rameau (and almost all theorists that have followed him since) faced severe problems when dealing with the minor tonality, he devised a basically ‘dualistic’ model to account for both harmonic major and harmonic minor. While some believe ‘harmonic dualism’ is obsolete and dead, the problems it was devised to solve unfortunately are not. In the third of the case studies, some steps in the development of harmonic dualism are retraced in brief, in order to illustrate certain problems. In each of the three case studies, I have added a few measurements and data to my arguments where appropriate to underpin the idea that empiricism might go well along with ‘theory,’ and that some of the pertinent problems should be restudied including experimental methodology.

3. Case Studies: from Vicentino to Riemann

3.1 *Chromaticism, tunings, intonation*

One can study the relation of theory to practice, for example, in the area of chromatic and enharmonic tone systems, scales, and intonation. It has been asserted more than once that much of the chromaticism found in works of the 16th and 17th century was a regionally rather limited and historically short-lived stylistic trend, and that the usage of enharmonic features was even more restricted because, on the one hand, instruments

64 For a comprehensive survey, see Braun 1994.

65 See Cohen 1984.

66 See Palisca 2006, and below in this text.

suited to perform such music would have been extremely rare, and, on the other, vocalists would hardly be capable to master the subtle intonation differences implied by tonal structures found in, for example, Gesualdo's later madrigals (of his 5th and 6th books of madrigals). In short, it was held that such music was a matter of 'speculative theory' rather than relevant to practice.

Meanwhile, the whole issue of chromaticism and ultra-chromaticism as well as enharmonic experimentation has been subjected to scrutiny including reconstruction of instruments, performance practice, and empirical investigation.⁶⁷ As to chromaticism, sources from the Faenza codex onward allow one to reveal tonal structures that can be systematized in a rational way.⁶⁸ Systems of tuning and temperament from the Middle Ages well into 'modern' times have been carefully investigated, and have been given precise mathematical description.⁶⁹ Also, it has been shown that the occurrence of consonances and dissonances in musical compositions of the 16th–18th centuries is not independent of the tuning systems in use at the time (Pythagorean, Just, Meantone⁷⁰). For example, keyboard adaptations of Dowland's *Lachrimae Pavan* (1604 or earlier) as elaborated by J. P. Sweelinck and M. Schildt, respectively, as well as many works of the time (like the *Galiarda Dolorosa* of Peter Phillips) require a meantone tuning, which offers as much as eight pure major thirds 5/4. The harmonic structure of those works can only be realized with a tuning that allows to preserve the 'sweetness' of simultaneous thirds and sixths. In this respect, the correct tuning certainly is an intrinsic part of the work in question, and not just an attribute from the exterior. Listening to the *Paduana Lachrymae* or to other works evidently written to be performed in specific tunings and temperaments probably is enough to understand how problematic is the view (motivated by aesthetics), according to which the 'intentional' (compositional structure represented as a score based on symbolic notation) is the 'interior' of music that is of relevance to music theory, whereas the 'acoustical exterior' (actual realization by means of sounds organized according to tone systems and tunings) is of but marginal interest.⁷¹ One can of course distinguish syntactic structure as put to paper and musical execution of a given work resulting in sound; however, the juxtaposition of 'interior' and 'exterior' (or, likewise, 'intentional' versus 'accidental,' 'compositional' versus 'acoustical,' 'historical' versus 'natural,' etc.) is not quite plausible when viewed from a musical perspective. Still far in the 19th century, Anton Bruckner, famous as a composer, recognized that compositional structures (such as modulations between distant keys) should be executed in a musically differentiated way (namely, in just intonation⁷²).

67 For a comprehensive survey, see Barbieri 2008 and also the Proceedings of the colloquium *Chromatic and enharmonic Music and musical instruments in the 16th and 17th centuries* (= Schweizer Jahrbuch für Musikwissenschaft 22). Bern: P. Lang 2003.

68 See Lindley 1994.

69 For example, Lindley and Turner-Smith 1993.

70 See Blackwood 1985.

71 For a critical examination of such views, see Schneider 1984, 1986.

72 See Strasser 2008, 134 ff.

Microtonal systems, as they emerged in the course of the 16th century, became executable both on a variety of instruments as well as for vocalists. Music written by Nicola Vicentino (1555) that comprises enharmonic tone steps in melody and some uncommon simultaneous intervals can be alternatively performed in the diatonic, in the chromatic, or in the enharmonic genus.⁷³ The latter two of course require professional singers (unspoiled by vibrato techniques that are common use among bel canto singers) and constant training. One can check the quality of intonation either by ear or by empirical measurement based on digital signal processing.⁷⁴ To demonstrate how subtle intonation in fact can be, an analysis of the final chord of the madrigal *O dolorosa gioia* (from Gesualdo's *Libro V*) as performed by *The Consort of Musicke* (London) is given.⁷⁵ The final G-major chord, sung on the syllable *vo* (... *e vi-vo*), consists of the notes *G-g-d¹-g¹-b¹*. The overall pitch pattern realized by the five voices can be seen when tracking their fundamental frequencies, which was done by a LPC algorithm.⁷⁶

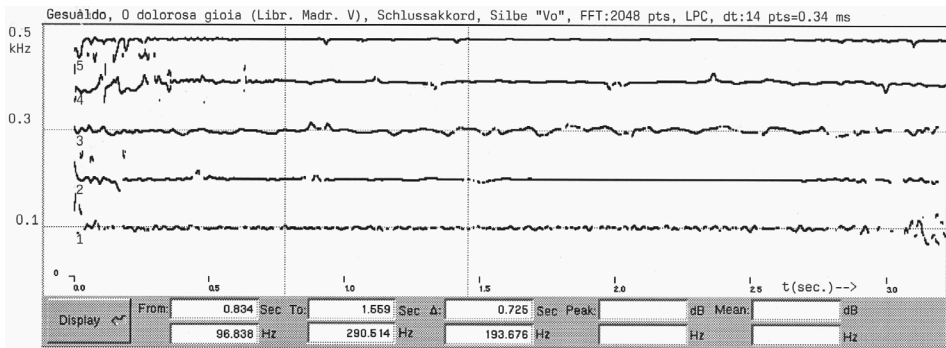


Figure 2. LPC analysis of final chord, *O dolorosa gioia*, *The Consort of Musicke*

According to the sound analysis, the soprano, the tenor and also the bass are exceptionally stable in their intonation once they have matched their voices to the fundamental frequencies required for the G-major chord. Looking into details of pitch adjustments one finds, for example, the bass succeeds in producing a stable fundamental frequency (of the note *G*) for ca. 1.5 seconds after some initial fluctuation. Bass, tenor and soprano in this chord realize fundamental frequencies representing just intonation pitches in a perfect manner (the other two voices being a few cents sharp).

Recordings by singers and ensembles trained to perform in highly chromatic or even enharmonic genres⁷⁷ demonstrate that intonation according to an elaborate scale and

73 See Cordes 2007, who offers a CD with his book that contains some impressive musical demonstrations, for example of *Dolce mio ben* sung and played “in genere chromatica & enharmonico.”

74 See Schneider 1998, Thies 1998.

75 Musical director: Anthony Rooley. CD: Éditions de l’oiseau lyre 410128-2 [Decca] 1984.

76 LPC = Linear Predictive Coding. As to details of signal processing applied to musical sound, see Schneider 1997a, 1998, 2000.

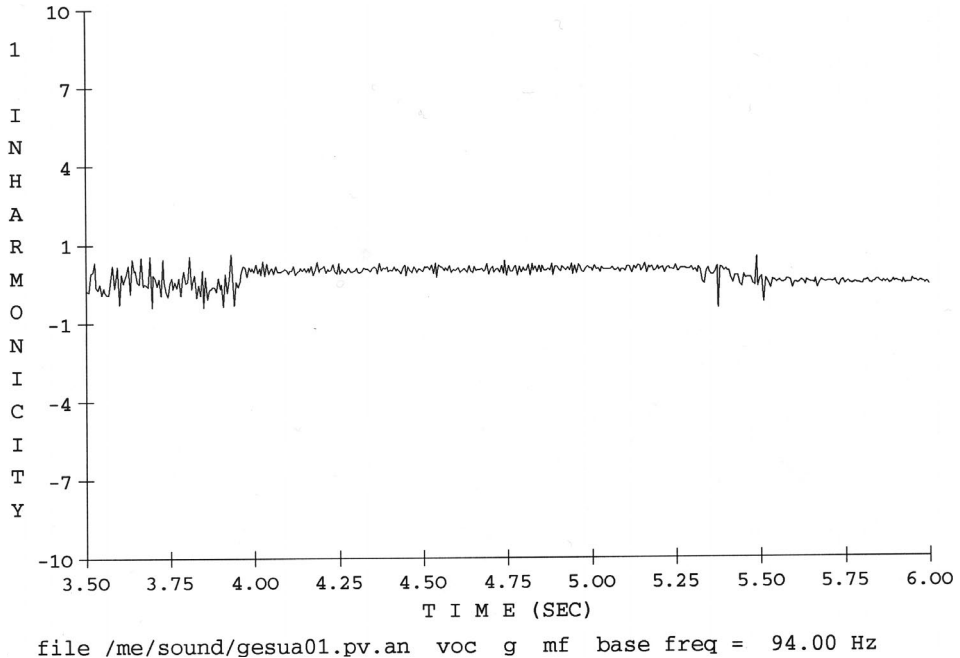


Figure 3. Tracking of the bass voice in the final chord of *O dolorosa gioia*

chord structure is possible. However, the singer must know which pitch representing a certain note she or he is expected to realize is the correct one in a given structure of intervals, chords and sonorities.⁷⁸ Whereas the intonation of the final chord in *O dolorosa gioia*, or in almost any other work, will hardly pose a problem as to the required pitches and respective fundamental frequencies, intonation patterns for chords and other sonorities found in works of music must be determined by analysis that, as a matter of fact, has to precede performance. In this respect, one needs to study, depending on the genre and style of the music in question, both the single voices and the relations they form in regard to ‘concordant’ and ‘discordant’ intervals⁷⁹ and the ‘vertical’ chord structure that emanates from (in particular, rhythmically conjoint) voices sounding simultaneously. Depending on a weighting of the two factors (counterpoint and harmony), different solutions in regard to intonation for a certain piece (or of some part of a piece) are feasible. One has to compare different solutions by checking their musical ‘plausibility’ as well as their sonorous coherence; this of course requires that different solutions must be realized

77 Besides vocal polyphony of the 16th and 17th centuries, respectively, one has to take into account also vocal music of the 19th and 20th century including microtonal compositions.

78 The German term ‘Zusammenklänge’ includes quite many (actual or possible) phenomena, e.g., harmonic and inharmonic intervals (see Dahlhaus 1975) formed by pairs of tones as well as more complex combinations of tones to be played simultaneously (see Helmholtz 1863).

79 See Dahlhaus 1975.

in sound either on conventional musical instruments (including the singing voice), or with the aid of computers and synthesizers.⁸⁰

Given that intervallic and harmonic analysis in regard to tone systems and tunings is necessary,⁸¹ and prerequisite to actually tuning an instrument (like for example a harpsichord) as well as to the performance of music, one could speak of a ‘primacy’ of music theory over practice. Such a point of view, which has been expressed by musically active theorists (from Zarlino to Karg-Elert) as well as by musicologists,⁸² may be disliked by those who believe music theory should stick to a descriptive and hermeneutic orientation rather than being ‘regulative’ in whatever way. One should bear in mind, though, that the very notion of ‘theory’ implies a claim to truth and validity besides objectivity and methodological coherence. Therefore, ‘music theory’ cannot abstain from such claims that are basic to all science. Moreover, music theory, beginning perhaps in the Renaissance era, often included a developmental momentum in that theorists designed systemic features and techniques they thought could or should be relevant for contemporary as well as for future composers when creating music.⁸³ Their approach thus was prospective rather than retrospective.

3.2 Rameau, ‘physicalism,’ and the minor tonality

Rameau has been criticized, more than once, for having introduced ‘physicalism’ into music theory⁸⁴ when trying to base his view of harmony on findings reported from acoustical investigations Sauveur and others had carried out on vibrating strings. Any critical examination of Rameau’s approach to harmony should take into account that, by about 1700–1720, research into acoustics and also matters pertaining to auditory perception had been done for almost 200 years.⁸⁵ For example, the phenomenon of resonance was certainly known since about 1500–1530, and in the 17th century there were various instruments in use employing sympathetic strings such as the ‘trompette marine’ and the viola d’amore.⁸⁶ Acoustic production of harmonics had come massively into play already around 1500 when organ building in many regions had begun to involve reed pipe stops plus compound mixture stops such as ‘Régale,’ ‘Dulzian’ and ‘Krummhorn,’ on the one hand, and ‘Zimbel,’ ‘Furniture’ or ‘Scharf,’ on the other.⁸⁷ Werckmeister explicitly names the odd harmonics that can be produced by overblowing a stopped flue pipe.⁸⁸

80 See sound examples provided by Sethares 1998, Cordes 2007, Barbieri 2008. Also material available on the Internet, e. g., the article of Olivier Bettens on Renaissance ‘Just Intonation’ offering instructive sound examples, see www.medieval.org/emfaq/zarlino/article1.html.

81 See Blackwood 1985, Vogel 1984, Lindley and Turner-Smith 1993.

82 See Riemann 1877, Vogel 1993.

83 This consideration is still found in the 20th century, in works on microtonal systems. See for example Habá 1927, Partch 1949, or Fokker 1966.

84 See Handschin 1948.

85 See Truesdell 1960, Cohen 1984, Crombie 1990, 365 ff.

86 See Külmer 1986.

87 See Klotz 1975, chapter VIII.

At the time Rameau was writing the *Traité*, various publications dealing with acoustics and sensation had become available. In regard to France, one of course thinks of Mersenne's⁸⁹ and Sauveur's writings. Rameau's *Traité* still refers to string divisions and proportions (as was customary⁹⁰); acoustics do not yet play a major role (there is the occasional observation, e.g. overblowing a flute into the octave "ne dépend que la force du vent"). However, Rameau points to the fact that string divisions lead to harmonic ratios representing perfect major ("... la corde entiere, qui renferme dans ses premieres divisions, des consonances qui toutes ensemble forment une Harmonie parfaite" (namely octave, fifth and major third).⁹¹ By 1701, Sauveur had published a comprehensive treatise on scales, intervals, and tuning that probably stems from a series of lectures he had delivered to the Académie Royale, in 1696–97.⁹² In this treatise, aptly titled *Système général des intervalles des sons ...*, Sauveur points to harmonics as found in string vibration but also relevant to explain the scale of tones available on natural horns (e.g., the cor du chasse).

As has been discussed elsewhere,⁹³ Rameau may have been informed about Sauveur's work by L. B. Castel before writing his *Nouveau système de musique théorique ...* (1726); in this book, Sauveur is quoted in particular in regard to aspects of tuning. From 1726 on, and clearly in the *Génération harmonique* (1737), Rameau had absorbed some of the subject matter discussed in the up-and-coming field of acoustics, and had made use of it for music theory. Parallels between the harmonic ratios of tones making up a major chord, and a series of harmonic partials as found in a single vibrating string are all too obvious. Both will give rise to a sensation of consonance (though the quality may vary somehow). A single thin brass string of a harpsichord when plucked contains many harmonics. The spectrum (Figure 4) taken from the sound of the lowest string of a French harpsichord⁹⁴ offers no less than 19 strong harmonics in the frequency range from 50 Hz to 1 kHz. A trained musician like Rameau, when analyzing the sound radiated from such an instrument, may well detect a number of the low harmonics that can be resolved by the ear.

Rameau's view of harmony, based on his own experience as a musician as well as on findings reported by Sauveur, was by no means unreasonable. The fact that a series of harmonics (as observed in a vibrating string) results in a sound which listeners experience as consonant, follows from the spectral harmonicity, which in turn implies strict

88 Werckmeister 1691, 23 f.

89 Mersenne 1636.

90 See Zarlino 1558.

91 Rameau 1722, 49 ff.

92 The treatise actually appeared in print in 1704 as part of the *Memoires* of the Royal Academy; it is similar in content to a *Traité de la théorie de la musique par M. Sauveur – 1697* preserved as a manuscript that seems to contain the material Sauveur had presented as lectures. See the Introduction of Rasch in (Sauveur 1984).

93 See Erwin Jacobi's Introduction to the reprint edition of Rameau 1726 (= Jean Philippe Rameau, *Complete Theoretical Writings*, vol. II, Dallas 1967, p. XV), Pischner 1963 and Christensen 1993.

94 The spectrum was taken from the C string (tuned to 52.3 Hz fundamental frequency) of a French harpsichord built by Nicolas Pigalle (Dijon 1771, see Beurmann and Schneider 2008).

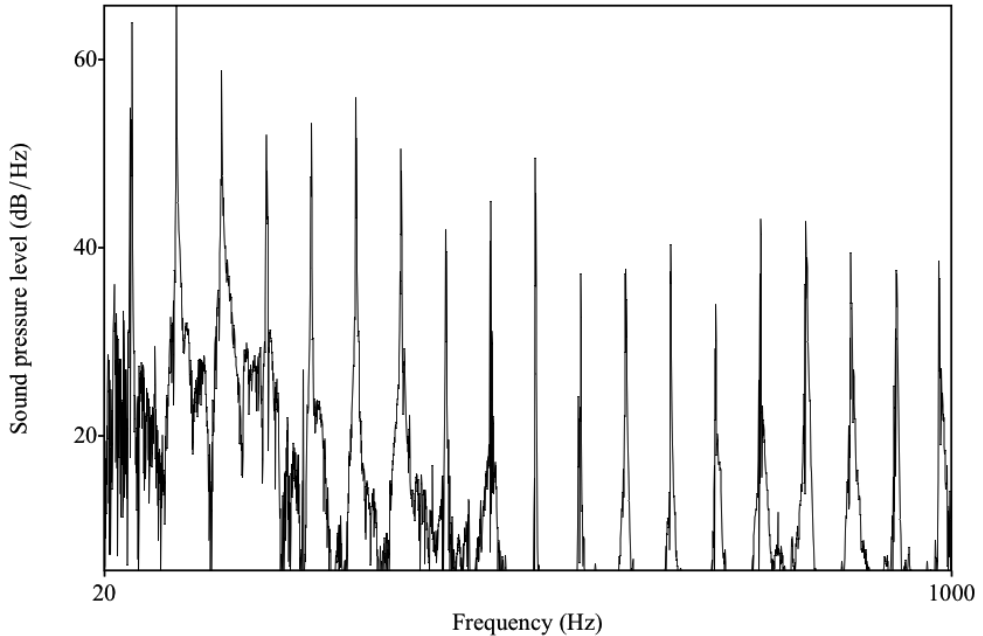


Figure 4. Spectrum, sound of the note/string C, Pigalle harpsichord (1771)

periodicity of the sound signal in the time domain. This in turn results in a periodic neural excitation pattern, which, finally, results in a sensation of consonance.⁹⁵ Of course, Rameau could not know all these interrelations then. However, one should accept that the ‘Physicalism’ introduced into music theory by Rameau and others (among them Mersenne, Huygens and Tartini) had an objective basis, on the one hand, and did fit into contemporary scientific discovery, on the other. The criticism levelled against Rameau and the “Obertöner” (as Handschin used to call them) in this respect seems disproportionate.⁹⁶ Moreover, there have been attempts recently to give music theory not only mathematical but also psychoacoustical foundations.⁹⁷ In particular the concept of consonance has been elaborated from this perspective,⁹⁸ in which some of Rameau’s ideas have been taken up and given a more formal as well as empirical treatment.

95 For examples, see Schneider 1997a, 1997b; Schneider and Frieler 2009.

96 See Handschin 1948, 178 ff. Handschin, one of the most thoughtful and erudite musicologists ever, was familiar with publications on perception and music psychology of his time. Though he discusses relevant theories and findings (for example, Helmholtz, Stumpf, Riemann, Lipps) in his *Toncharakter*, Handschin’s own perspective was more philosophical than empirical (one can see traces of Platonism in his derivation of tonal relations).

97 See Terhardt 1986, Parncutt 1989.

98 See Terhardt 1998, 397 ff.; Schneider 1997a, 1997b; Schneider and Frieler 2009.

The problem Rameau and legions of later music theorists were faced with is the minor tonality, for which no easy acoustical explanation is at hand.⁹⁹ One of the remedies Rameau employed (and continued to put forward even in his late works¹⁰⁰), is a symmetrical scheme of “aliquotes ou sous-multiples” and “aliquantes ou multiples.” Central to Rameau’s considerations is a symmetry between the harmonic proportion $1 : 1/3 : 1/5$, and the arithmetic proportion $1 : 3 : 5$. Basic intervals derived from the proportions $30 : 24 : 20$ and $30 : 25 : 20$ (Rameau 1726, 21) yield the major and the minor chord, defined by their “Son fondamental, Quinte, Tierce majeure & mineure.” Rameau, to be sure, considers the two thirds to have equal weight since to him the pure fifth and the two thirds are “Consonances premieres & directes.” Their counterparts, “la Quarte, la Sixte mineure & la majeure,” he labels “indirectes ou renversées.” (The term “renversement” is used by Rameau also with respect to inversions of chords). Assuming ‘octave equivalence’ because “... un son quelconque est toujours sous-entendu dans son octave ...,”¹⁰¹ and applying further a reduction whereby intervals are all put into one octave (instead of the three needed to locate these intervals on a single string¹⁰²), Rameau developed a neat scheme of the musical scale as well as of the intervals constitutive for harmony. However, whereas the harmonic proportion actually fits the frequency ratios of partials in a string or air column (and hence can be regarded as given by nature), the arithmetic proportion, which may also be applied to the division of a string, normally will not result in ‘sub-harmonics’ (see below). It was during the 18th century that string vibration was investigated from both a mathematical and experimental perspective; the results demonstrate that a string set to longitudinal and transversal vibrations produces a series of harmonics above a fundamental. Since the phenomenon of resonance was essential in both Sauveur’s and Rameau’s reasoning, and since the latter had employed the arithmetic proportion to establish the minor third (as well as the minor chord), one would have to see whether a string could resonate so as to produce those ‘aliquantes.’ Both Rameau¹⁰³ and d’Alembert¹⁰⁴ took pains to find at least faint ‘tremblements’ at the unison; Rameau had to admit, however, that there was no such acoustic support for harmonic minor as there was for major. This led him to conclude that harmonic minor, though basically on a par with harmonic major, is not just as natural as major, on the one hand, but – provided one follows Rameau’s derivation – offers many options, on the other (“... le mineur y sera susceptible d’une bien plus grande variété que le majeur”).¹⁰⁵ The other aspect suited to connect a major chord to its parallel minor according to Rameau is that they have two tones in common (“... le générateur du majeur, & sa tierce,

99 As to Rameau, see Pischner 1963, 101 ff.; Christensen 1993, chapters 6 and 7.

100 For example, Rameau 1760, 193 ff.

101 Rameau 1722, 8.

102 Rameau 1722, 15.

103 Rameau 1750, 63 ff.

104 d’Alembert 1757, 10, 39.

105 Rameau 1750, 75. This follows of course if one uses the ‘natural’ or ‘pure’ minor scale (the Aeolian mode) along with ‘harmonic minor’ (incorporating a ‘note sensible’) and ‘melodic minor’ (6th and 7th scale step raised by a minor second).

forment la Tierce & et la Quinte du générateur du mineur ..."). A similar argument, to be sure, was brought forward by Helmholtz when he said that chords are closely related to each other if they have one or even two tones in common (for example, C-major = $c-e-g$ and a-minor = $\underline{a}-c-e$).¹⁰⁶

3.3 Rameau to Riemann: the emergence of 'harmonic dualism'

One has to see that particularly the fundamentals of Rameau's theory of harmony are close to what later, in the 19th century, emerged as 'harmonic dualism.' Since this term nowadays is treated by some as if standing for an obsolete "attempt to answer a question that no longer interests us,"¹⁰⁷ it seems necessary to reconstruct basic considerations that led to 'harmonic dualism.' Perhaps the most basic, and one certainly still of interest to musicology, is that music theorists for centuries struggled to 'explain' harmonic minor (and the minor tonality in general) as a tonal and perceptual framework that differs in certain respects from major. Harmonic minor is found in so many works of the 17th and early 18th centuries, respectively, that theorists had to deal with minor chords and the minor tonality in a reasonable way. In general, theorists¹⁰⁸ elaborated on musically usable consonant and dissonant intervals before discussing chords and chord progressions.¹⁰⁹ Mattheson relates seven diatonic "Klang-Stuffen" (scale steps from d' to c'') to 'Tonarten' in that he proposes a symmetrical construct, which results in minor thirds above d' , e' , a' , and b' , and major thirds above f' , g' and c'' .¹¹⁰ If one adds, in addition to the major or minor thirds, a fifth to each of the diatonic "Klang-Stuffen," the three major chords of c , f and g would form a cadence in major, and the minor chords of a , d and e correspondingly a minor cadence. Mattheson believed that there is a 'natural order' according to which a "soft" or, alternatively, a "hard" third must be assigned to a given diatonic scale tone (one obvious reason is to avoid accidentals and alterations).

Tartini (1754), one of the pioneers in research on combination tones, saw that the minor tonality was cumbersome to derive from the series of harmonic partials, and also less coherent than major when viewed from the combination tone perspective. His solution, based on elaborate geometric operations, indeed leads to a minor chord, in which the tones $C-E^b-G-c$ stand in the relation $12:10:8:6$.¹¹¹ The minor third of course is the arithmetic mean of the tones C and G , that is, Tartini applies an arithmetic division to the fifth to yield the minor third (and a harmonic division to obtain the major third). Though Tartini's approach (based on two geometric figures: the 'harmonic' circle and the 'arithmetic' square) is his own, it reminds of Zarlino (and also of Rameau) since the harmonic and the arithmetic division of the fifth finally yield the major and the minor chord, respectively.

¹⁰⁶ Helmholtz 1863 (6th ed. 1913, 481).

¹⁰⁷ Rehding 2003, 32.

¹⁰⁸ For example, Zarlino 1558, Rameau 1722.

¹⁰⁹ See, for example, Heinichen 1728, 106 ff.

¹¹⁰ Mattheson 1739, Vorrede, IV.

¹¹¹ Tartini 1754, 172 ff.

Zarlino had discussed arithmetic and harmonic division of intervals, which can be ordered in reciprocal series.¹¹² In c. 31 of the third part of his *Istitutioni*, he dwelled on how musical intervals could be combined in settings comprising several voices.¹¹³ He gave a description saying that the range of concords not only consists of consonant intervals (forming two voices), but also depends on the position "... di una chorda mezzana, che si pone tra la Quinta nella compositione."¹¹⁴ The third voice or string that lies within a fifth divides this interval in two possible ways, in that it is either divided "... o mediacione Arithmetica or alla mediocrità Harmonica; & da questa varietà dipende tutta la diversità, e la perfettione delle Harmonie ..." Though one may dispute the exact meaning of certain words or even of parts of phrases found in this chapter, it seems clear that Zarlino saw the major and minor third as effecting and representing the harmonic and arithmetic division, respectively.¹¹⁵ Zarlino further distinguishes the two thirds with respect to the affective power of music as serene ('allegra') and sad ('mesta'), respectively.

Riemann, translating and interpreting Parte II, c. 31 as well as some related paragraphs of the *Istitutioni*,¹¹⁶ and pointing also to chapters of the *Dimostrazioni harmoniche* (1571), came to the conclusion that Zarlino, by employing the harmonic and the arithmetic progression to form the relevant intervals, had at the same time determined the major and the minor chord as the two fundamental harmonic elements. Riemann's interpretation has been questioned by Dahlhaus (1957) who showed that Riemann's translation does not exactly match Zarlino's text. In fact, there are some debatable sentences (including gaps) in the translation Riemann provided; seeing an important historical source for 'harmonic dualism,' he apparently strived to render an interpretation of Zarlino's statements consistent with his own view. Though Riemann's reading of Zarlino may fall short of philological exactness,¹¹⁷ it would be unjust to denounce his translation as perfunctory, or even fabricated. Zarlino apparently distinguished the major and the minor genus according to the two progressions (see above) that resurfaced so prominently in Rameau's writings. Rameau in fact was expanding on Zarlino (notwithstanding harsh criticism found in Rameau 1722) when using the harmonic and arithmetic progression as a mathematical (that is, 'symbolic') approach to the generation of musical intervals. However, Rameau also saw the reciprocal series derived from the respective proportions as a physical model until he became aware, around 1749/50, that a generation of the minor triad from acoustical principles was difficult to develop, and that the arithmetic progression applied to string lengths could not be employed along with theories of sympathetic resonance in a useful way.¹¹⁸

112 Zarlino 1558, I, c. 36–40; see Fend 1989, 434.

113 "... gli intervalli relati nelle compositioni di piu voci" (Zarlino 1558, 181. 3rd edition 1573, 210).

114 Zarlino 1558, 181.

115 "... prima posto la Terza maggiore, che faccia la mediacione Harmonica; poniamo dapoi la minore, che farà la divisione Arithmetica." (Zarlino 1558, 3rd edition 1573, 211).

116 Riemann 1920, 389 ff.

117 Riemann (1920) of course translated only those portions he believed to contain the most relevant information with respect to harmony.

118 See Rameau 1750, 63–83; Christensen 1993, chapter 6.

In principle, Rameau had set up a ‘dualistic’ theory of harmony notwithstanding the problems mentioned. Harmony became a central area of music theory consequent to developments in music that led to writing homophonic textures where the ‘vertical’ dimension grew almost independent of the ‘horizontal.’ Of course, there is always an interplay between voice-leading (voicing) and chord progressions, but even prominent music theorists have tended to focus on one of the two dimensions, with a relative neglect of the other.¹¹⁹ The theory of ‘harmonic dualism’ does not assume that the ‘vertical’ dimension of chords and chord progressions can be modelled and analyzed without regard for voice-leading; however, in harmony (as treated in Riemann 1893, Karg-Elert 1930) and in ‘harmonic analysis’ (as explained in Vogel 1975, 1984, 1993) chords and chord progressions are usually given more weight than are aspects of voice-leading and melody.¹²⁰

After Rameau had attempted to build harmony on acoustic foundations, Hauptmann (1853) took a different approach, in which acoustics played but a small role; there is the occasional hint to the “natural unlimited series of tones ordered according to ratios of vibration,”¹²¹ but Hauptmann pursues rational musical discourse rather than scientific demonstration.¹²² There are speculative ideas in Hauptmann due to his dialectic constructs, however, he tries to qualify his approach as logical and ‘natural’ because of corresponding to human apperception.¹²³ Hence Hauptmann believed he could claim that there are three, and only three ‘directly apprehensible intervals’ (octave, pure fifth, (major) third).¹²⁴ The same basic intervals occurred in Rameau, however, for acoustical reasons. Hauptmann shared some of Rameau’s ideas concerning the minor tonality (namely that a major chord and its parallel minor have two tones in common), and there is also the distinction of a tone “having” a third and a fifth, and a tone that “is” either a major third or a fifth.¹²⁵ The difference between ‘having’ and ‘being’ (German: *Sein und Haben*) rose to prominence in von Oettingen’s dualistic approach in defining major and minor chords relative to the structure and weight of their harmonic partials.¹²⁶ Oettingen (1904), in describing harmonic analysis and synthesis of partials to explain the principles of ‘Tonicität’ and ‘Phonicität,’ respectively, put the terms back on their feet, so to speak, when demonstrating that the acoustic and perceptual coherence of a pure minor chord is by no means inferior to that of a major chord since in both there are partials coinciding in frequency as well as partials with frequencies an octave apart. Coinciding partials as well as the marked periodicity of the resulting waveshape are acoustical correlates to

119 See Federhofer 1981.

120 The Schenkerian approach presumably accounts for both the ‘horizontal’ and the ‘vertical’ dimension in an adequate way (see Federhofer 1981). However, Schenker’s view of harmony is based on diatonic ‘Stufenlehre,’ which is not suited to deal with complex harmonic patterns such as found in many works of the 19th and early 20th centuries, respectively.

121 Hauptmann 1853, 33.

122 See Rummenhüller 1963.

123 Hauptmann 1873, 4 ff.

124 Hauptmann 1873, 19.

125 Hauptmann 1853, 32.

126 See Oettingen 1902–06, 1913.

a percept characterized as ‘Verschmelzung.’¹²⁷ Oettingen’s considerations, which combine some of the acoustical analyses of Helmholtz with psychological inferences given by Stumpf (1898), can be summed up in a table. For each tone in a pure major and its parallel pure minor chord, partials 1–10 are considered (Oettingen’s own concept did span four octaves):

Tonic triad c – e – g	Phonic triad <u>a</u> – c – e
{c → e'''}+{e → <u>g#'''</u> }+{g → b'''} }	{ <u>a</u> → <u>c#'''</u> }+{c' → e'''' }+{e' → <u>g#''''</u> }
Ratio 4 : 5 : 6	Ratio 6 : 5 : 4
Major chord: 4 frequency-coinciding partials	Minor chord: 5 frequency-coinciding partials
plus 4 octave-related partials	plus 5 octave-related partials

Table 2. Acoustical structure of a pure major and its parallel minor chord

Taking into account chords composed in this manner (table 2), and applying pitch relations in just intonation (as is mandatory according to the concepts of Hauptmann, Helmholtz, Oettingen, Riemann, and also Stumpf), it is straightforward to synthesize such chords that, given their temporal and spectral composition, will result in highly ‘harmonic’ percepts since in particular the pure major chord exhibits spectral fusion that translates into a strictly periodic wavetrain where the period is clearly determined by the common denominator of the fundamental frequencies of the three complex tones forming a chord. Since, in our example, these frequencies are {244, 305, 366 Hz} = 4:5:6, it follows that $f_0 = 61$ Hz, and $T = 1/f \approx 16.4\text{ms}$ ¹²⁸. Hence, the major chord is perceptually reinforced by a virtual pitch that can be taken as a ‘basse fondamentale.’¹²⁹

For the a-minor chord whose complex tones have fundamental frequencies at {203.333, 244, 305 Hz}, the appropriate virtual pitch is not just as easy to determine.¹³⁰ By calculation and measurement,¹³¹ one can estimate a virtual pitch just above 20 Hz. Similarly, a c-minor chord with fundamental frequencies of {244, 292.8, 305 Hz} yields a virtual pitch at ca. 25 Hz. The virtual pitches found for the major and the two minor chords might be of different salience.¹³²

127 See Schneider 1997a, 1997b, and Schneider and Frieler 2009.

128 Fourier synthesis was done with Mathematica 7.1. Fundamental frequency for the C-major chord is $c' = 244$ Hz, and $a = 203.333$ Hz for the a-minor chord. Amplitudes of the first ten partials per note are defined as $A = 1/n$, $n = 1, 2, 3, \dots, 10$.

129 See Terhardt 1982.

130 For a critical account of root determination concepts, see Thomson 1993.

131 Models included autocorrelation and subharmonic matching (see Schneider and Frieler 2009).

132 Depending on the so-called ‘dominance region’ for virtual pitch, see Terhardt 1998, 349f.

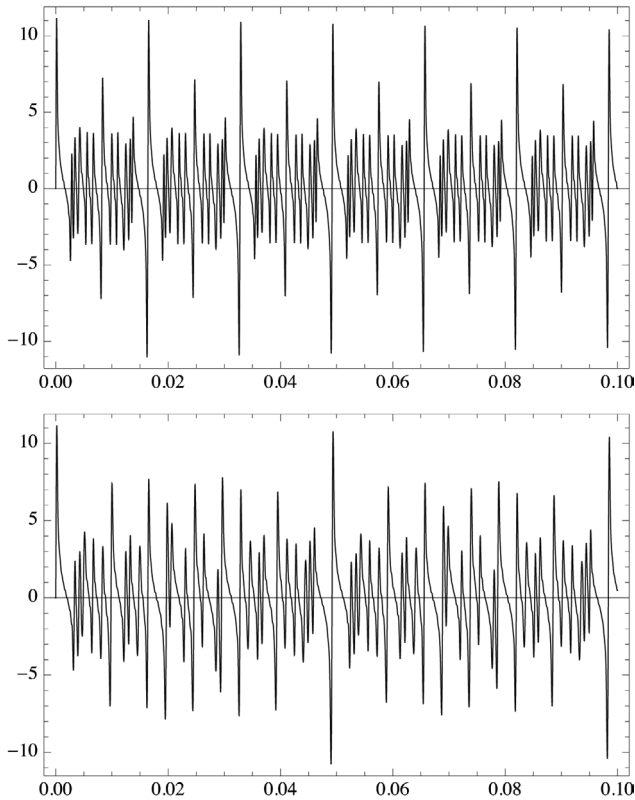


Figure 5. Synthesis of C-major (upper) and a-minor chord (lower), ten partials per note (= 30 spectral components per chord), resulting waveshapes for $t = 0.1$ seconds

From these examples, it can be inferred that the difference between a major and a minor chord (realized in just intonation, and in 'root' position) is that the minor chord is likely to evoke a somewhat weaker low virtual pitch. With respect to another relevant perceptual feature, spectral brightness, major and minor chord based on identical fundamental frequency differ but little. For the C-major and the c-minor chord used as examples, the average spectral centroid is ca. 1550 Hz and ca. 1500 Hz, respectively. Normalized spectral centroid gives the brightness of a sound; as figures 7a/b demonstrate, both chords are almost identical in brightness.¹³³ This result does not support the commonly held view according to which a chord in minor is just the muddy variant of its major prototype.

In the light of the empirical data complementing theoretical considerations, Oettingen was certainly on the right track when he expressed the need to deal with harmonic minor as a phenomenon that cannot be simply taken as 'blurred major' ("getrübbtes Dur,"

¹³³ They are identical in level, which is 77.43 dB shortly after onset, and 60.5 dB after 3 seconds (the sounds have a damped temporal envelope defined by $E^{(-0.7t)}$). An interesting difference (to be reported elsewhere) becomes obvious in a formant analysis of the two sounds.

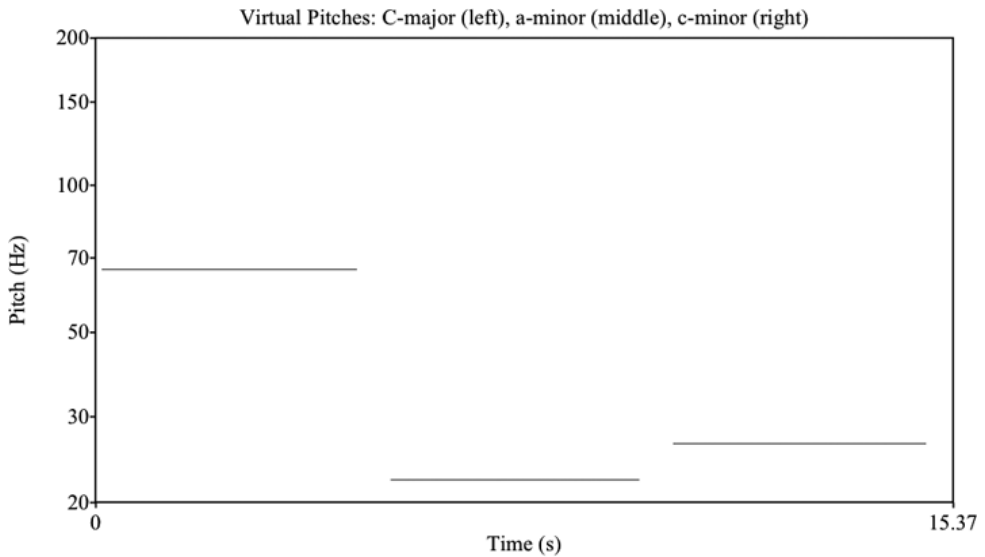


Figure 6. Virtual pitches found by autocorrelation algorithm for the C-major, the a-minor and the c-minor chord (61, 20, 25 Hz)

as Helmholtz, Hindemith and others saw it). Oettingen took great pains to reinstall music theory on a basis provided by musical acoustics and psychoacoustics.¹³⁴ To this aim, he developed special signatures indicating tone relations (that can be traced by means of a two-dimensional tone net), and he also conducted many experiments where listeners were requested to judge chordal patterns played in several intonation variants. For these experiments, he employed a special harmonium (the 'Orthotonophonium'¹³⁵) that offered 53 notes per octave (in another version, even 72 notes per octave). Oettingen's approach included empirical work along with theoretical argument; he devised a coherent and systematic treatment of harmony based on a strictly dualistic perspective (which led him to make the note *d* the central note for both the tonic and the phonic chord¹³⁶).

When Riemann, in 1874, began to publish on music theory, he could draw from Hauptmann (1853), Oettingen (1866), and Helmholtz (1863), as he in fact did in various contexts (approving as well as contradicting points of view issued by his masters).¹³⁷ For a time, he admired also Stumpf for his *Tonpsychologie* (1883/1890), and in particular for the concept of 'Verschmelzung' that is at the core of Stumpf's theory of consonance (1898).¹³⁸ Since it is not possible, at this place, to reconstruct Riemann's theoretical concepts in any detail (which he, moreover, changed considerably in the course of more than

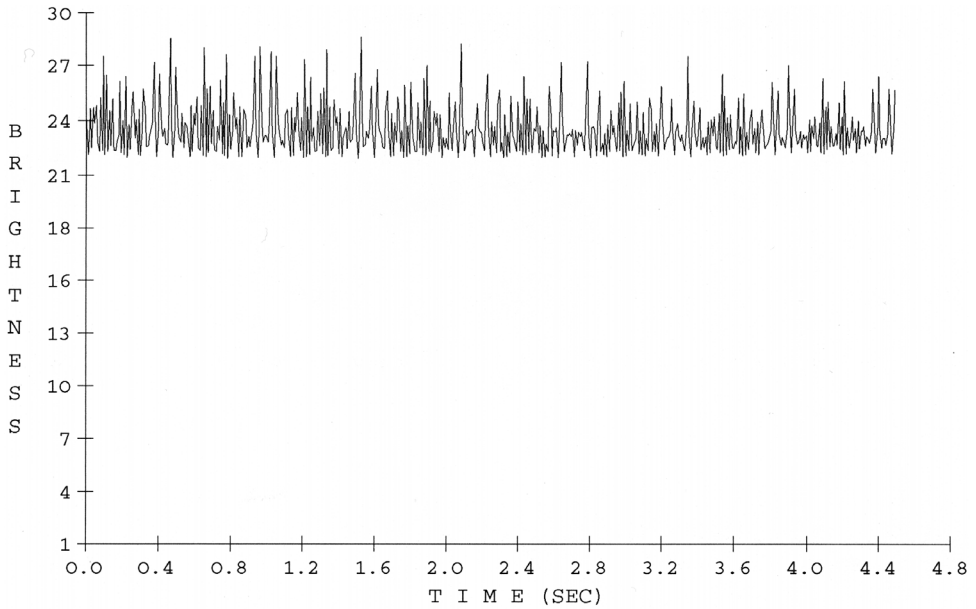
134 Oettingen 1866, 1913.

135 See Oettingen 1913, Goldbach 2009.

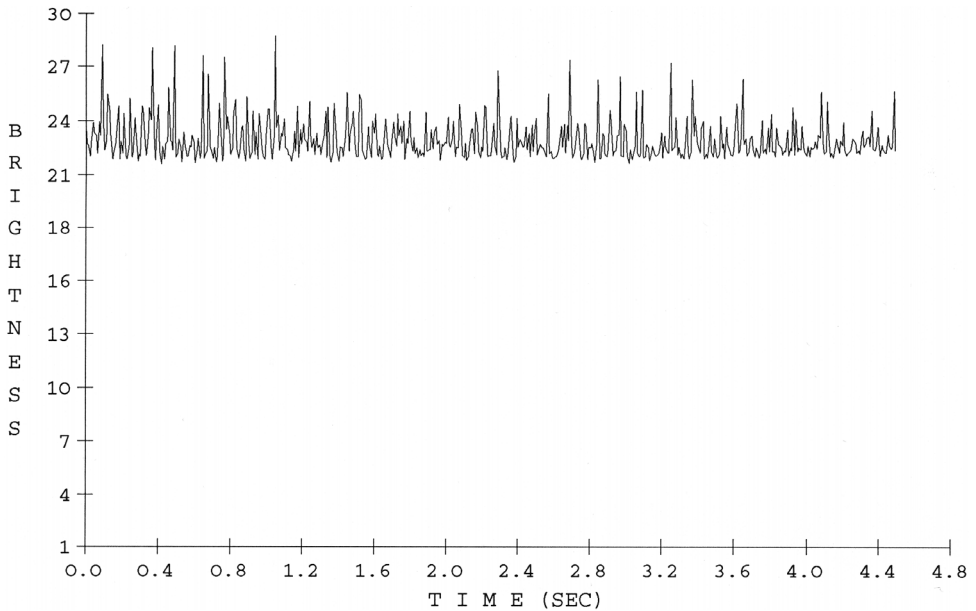
136 See Oettingen 1903, 64 ff.

137 A more detailed treatment of issues concerned in this chapter is given in Schneider 2008a.

138 Schneider 1997b.



file /me/CDursnd.pv.an math c mf base freq = 61.00 Hz
nhars = 393, t1 = 0.000, t2 = 4.492



file /me/cmoll.pv.an math c mf base freq = 61.00 Hz
nhars = 393, t1 = 0.000, t2 = 4.492

Figure 7. Brightness of a C-major and a c-minor chord, fundamental frequency = 244 Hz

four decades he remained active as a musicologist and author), we could simply point to Riemann's account of the history of music theory where he, at the very end, states that this book should be regarded, in particular in its third part (titled "Die Harmonielehre," Chapters 14–16), as a "Rechenschaftsbericht über die Herkunft meiner Ideen zur Theorie der Musik."¹³⁹ Indeed, Riemann saw himself as completing what Zarlino had begun, and what Rameau, Hauptmann, Oettingen, Helmholtz and others had tried to achieve: to ground music theory, and, most of all, harmony, on a firm, scientifically proven fundament. This fundament, however, did undergo significant changes in the course of Riemann's scientific career. He started with a perspective on 'Musikalische Logik' as outlined by Hauptmann (on the basis of 'harmonic dualism' and just intonation), but soon sought to integrate psychoacoustics of pitch, interval and chord perception as provided by Helmholtz. In certain respects, Oettingen offered a combination of both since he took acoustics into account (without overstressing acoustical principles and facts), and also saw the need to investigate the relationship ('Verwandtschaft') of chords from the point of musical perception and apperception. By the time Riemann wrote his dissertation under the guidance of Hermann Lotze (a physician and philosopher by academic training), he aimed at a (in modern terminology) 'transdisciplinary' approach to the study of music perception.¹⁴⁰ There are indications that Riemann sought to cooperate with scholars of the ranks of Helmholtz and Oettingen, for example, in clarifying the matter of audible undertones.¹⁴¹ In certain respects, he fell victim to Helmholtz' hypothesis according to which certain parts of the basilar membrane in the inner ear can be viewed as a system of parallel-wired mechanical resonators working like the strings of a piano.¹⁴²

With the anatomical knowledge at hand around 1855 (Alfonso Marchese Corti had just discovered what became known as Corti's organ), Helmholtz thought "... dass jedes solches Anhängselchen, ähnlich den Saiten des Claviers, auf einen Ton abgestimmt ist"¹⁴³ Riemann took up this resonance hypothesis, and even expanded it, in his doctoral dissertation¹⁴⁴ in order to find a reasonable explanation for the perception of harmonic minor. His claim was as follows: "... die den Untertönen eines angegebenen Tones entsprechenden Fasern der Membrana basilaris schwingen partiell mit und wir haben daher die Vorstellung der Untertöne implicite." To be sure, Riemann did not argue that undertones were present in the soundwave; he infers that the inner ear mechanism (not well understood yet in 1873) would bring about a sensation of undertones only 'implicite.'

139 Riemann 1920, 529.

140 Riemann 1874.

141 See the 'Beilage' in Riemann 1877, 121–123. See also Münnich 1909.

142 Helmholtz 1857, 1863. Helmholtz explained details of his hypothesis in the Beilage IX ("Schwingungen der Membrana basilaris der Schnecke") of his famous book. In order to follow Helmholtz' line of physical argument, one must have a certain understanding of mechanics (vibration of membranes, which involves Bessel functions, see Schneider 1997a, 201 ff.). Correct solutions relevant to inner ear physics were provided by Georg von Békésy (who was awarded the Nobel prize in medicine, in 1961) and others, not until the 20th century.

143 Helmholtz 1857, repr. 1971, 33.

144 See Riemann 1874, 6–13.

Riemann's various statements on undertones and resonance phenomena have been reiterated time and again, most often with the intention to discredit Riemann as well as harmonic dualism; the *communis opinio* is that if there are no undertones, there is no basis for dualism. Such a view is simplistic, though; neither did harmonic dualism as developed by Hauptmann and von Oettingen rest on assumptions which would require acoustically 'real' undertones, nor did dualistic concepts issued in the 20th century by Karg-Elert (1930) and his pupils as well as by Martin Vogel (1975; 1993) who perhaps remained the scholar closest to Oettingen's approach. Moreover, at least some of Riemann's experiments have been misinterpreted in order to ridicule him (and thereby also harmonic dualism; the strategy is all too obvious). Riemann evidently did not begin his inquiry with some fallacious "moonshine experiment,"¹⁴⁵ but had worked with his harmonium he had tuned to produce intervals that gave rise to sensation of a virtual pitch or pitches below the fundamental frequencies of the tones played. His observations, which can be interpreted in regard to combination tones as well as the 'missing fundamental' (Schouten's 'residuum') and subharmonic matching,¹⁴⁶ have been checked and accepted as valid.¹⁴⁷ It was probably from Helmholtz' (1863) treatment of resonance phenomena that Riemann set out to conduct some experiments where he believed to have detected 'undertones' in vibrations of undamped piano strings.¹⁴⁸ Most critics have categorically denied that Riemann could have heard what he claimed to have heard; however, Carl Stumpf, who repeated Riemann's experiments, came to the conclusion that Riemann in fact could have sensed certain sounds, which Stumpf attributed to the overall vibration pattern ("... allgemeine Erschütterung des Instruments ...") of the piano once set to motion.¹⁴⁹ Stumpf thought that Riemann was misled to take these sounds as 'undertones.' However, Stumpf found in some of his own experiments (simultaneously playing two strings of his violin) not only partial sympathetic vibration of the string tuned lower when bowing the string tuned higher, but also sympathetic vibration of the full string length when his bowing was strong. He concluded: "Wie leicht könnte man nun hieraus wieder auf die objective Existenz der Untertöne schließen!"

Apparently, things are not as simple as to dismiss the case altogether. If one repeats one of Riemann's 'moonshine' experiments (familiar to musicians and musicologists), it gives some cues that might help to explain Riemann's presumed sensations. In figure 8, the spectrum of a sound recorded from a Blüthner concert grand piano (280 cm) is shown.¹⁵⁰ The large peaks f_1 and f_2 at 129,25 and 258,71 Hz, respectively, are the first two harmonic partials of the note/tonc (C3) that was played while the keys C (C2) and

145 See Rehding 2003, chapter 1. The various meanings of 'moonshine,' to be sure, include nonsense or rubbish as well as home-made 'booze.'

146 See Schneider 1997a, Terhardt 1998.

147 Reinecke 1962.

148 See Riemann 1877, XIII, 5 ff.

149 Stumpf 1890, 264 ff.

150 Recording: January 18th, 2010, at the Musiksaal of the Institute of Musicology, University of Hamburg with one microphone (AKG C 414B-TL II, set to Cardioid) on DAT (48 kHz/16 bit linear). Spectral analysis parameters: FFT 131072 samples ($df = 0.366$ Hz), parabolic interpolation for frequency at peak reading, Blackman 4 weighting, zero-pad factor 2.0.

C, (C1) had been pressed before, and the dampers had been lifted from all the strings. The spectrum obviously contains (a) considerable energy below f_1 ; among the many components is one about one octave below f_1 . Also, there are (b) many components between f_1 and f_2 that are certainly not partials of the note c, which has been played, but partials of strings whose fundamental frequency is below that of the note c. Though the single string will not produce ‘undertones’ under normal conditions,¹⁵¹ sympathetic resonance phenomena such as recorded from undamped strings in a piano may have led Riemann to believe that he had detected ‘undertones.’

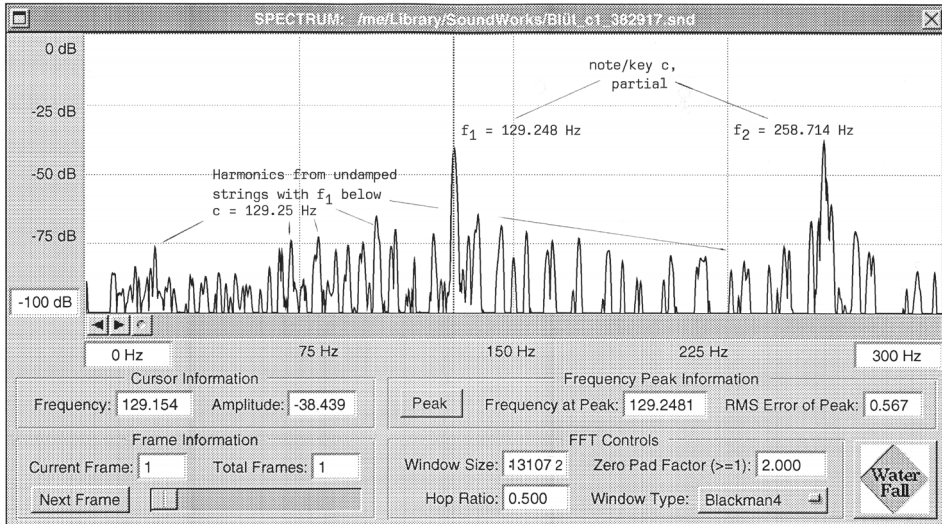


Figure 8. Spectrum (LTAS), grand piano, note/tone c plus low spectral components

Riemann’s failure to see his experimental observations acknowledged went along with another disappointment; the cooperation he seems to have sought with scientists and psychologists like Helmholtz, Oettingen and Stumpf didn’t materialize. One can see how Riemann’s attitude changed from youthful enthusiasm as expressed in his dissertation and in the *Musikalische Syntaxis* (1877) to defence and disappointment as expressed in his series of articles on ‘harmonic dualism’ (1905). In between, probably realizing that Oettingen’s strict approach to dualism was difficult to reconcile with ordinary listeners’ experience, and even more so with contemporary musical practice based on the piano and equal temperament, Riemann worked out a compromise.¹⁵² In essence, it asserted that harmonic dualism as well as just intonation is correct in theory, but not possible and not even needed in practice. This point of view is maintained in the *Ideen zu einer Lehre von den Tonvorstellungen* (1914–16). A central component of Riemann’s compromise

¹⁵¹ It is in fact possible to produce subharmonics in strings, by parametric excitation. See Meyer and Guicking 1974, 438.

¹⁵² Elaborated in Riemann 1895.

that he presented in several publications (with updates and revisions) was his doctrine of ‘enharmonic identification’; this notion turns up already in the *Syntaxis* (1877) and is elaborated in more detail in *Das chromatische Tonsystem* (1895). However, still in the *Ideen* the ‘enharmonische Identifikation’ is heralded as a new and groundbreaking concept Riemann promises to present in a detailed format in the future.¹⁵³ One can of course see what he meant by ‘enharmonische Identifikation’ as well as by a concept labelled ‘economy of imagining’ or conceiving (“Ökonomie des Vorstellens”).¹⁵⁴ Both would have needed detailed treatment since Riemann was calling for the most differentiated ‘Tonvorstellungen’ possible, on the one hand, and at the same time saw a need for reduction of such elaborate images as could arise from the multitude of tonal relations at hand in just intonation, on the other. ‘Economy of imagining/conceiving’ thus should regulate our ‘Tonvorstellungen’ in an effective way. However, elaboration and simplification can become conflicting strategies so that their possible interplay would have to be explained in a full-grown ‘Lehre von den Tonvorstellungen.’

Another simplification Riemann undertook concerned the two-dimensional *Tonnetz*. In Oettingen’s original publications,¹⁵⁵ the rectangular or quadratic tone-net relates eight tones to each center tone one may choose, namely the fifth below and above this tone, the major third above and below this tone, and the major third below and above the two fifths, respectively. Riemann had adopted this scheme while others had simplified it early on to six tone relations per centre tone.¹⁵⁶ In his *Ideen*, Riemann used the diamond-shaped tone-net that implies the reduced six-note format. It is this reduced format that has been adopted widely in “Neo-Riemannian” schools whereas Oettingen’s tone-net has been expanded into a three-dimensional structure incorporating tone relations based on prime numbers {3, 5, 7} by Vogel.¹⁵⁷

Whereas Oettingen did strive to (re)unite music theory and science, Riemann’s compromise concept separated music theory from musical acoustics and psychoacoustics,¹⁵⁸ to the effect that music theory is not concerned with musical sound organized so as to realize tonal images (‘Tonvorstellungen,’ which are also a central part of Oettingen’s concept), but with tonal images conceived apart from actual sound and intonation patterns. Already in the *Syntaxis*, Riemann’s argument is set forth in regard to reading music and correct orthography (making use of ## and bb), a perspective maintained in the *Ideen*.¹⁵⁹ Riemann’s ‘Musikalische Logik’ was enough to secure correct understanding of musical

153 See Schneider 2008a, 47. A point of interest is the date at which Riemann wrote this article that was published in 1914–16 but could have been drafted much earlier as it contains in fact ‘ideas’ Riemann had pondered for many years.

154 Riemann probably took this concept from the main reviewer of his thesis at Göttingen, Hermann Lotze (who uses the phrase “Oekonomie unseres Vorstellens” in his history of aesthetics in Germany, in a critical account of the psychology of Herbart; see Lotze 1868, 282 ff.). I will discuss this concept and its (Kantian, Herbartian, etc.) background in more detail elsewhere.

155 Oettingen 1866, 1903 ff.

156 See Oettingen’s complaints about this unduly change, 1903, 382 ff.

157 Vogel 1975, 1993.

158 See Riemann 1895, 1905, 1914–16.

159 Riemann 1877, 117–120.

structure, and of harmony in particular, notwithstanding abstruse deviations from just intonation pitch patterns that might occur in music performances. However, Riemann never gave up ‘harmonic dualism’ as a concept, nor did he relinquish just intonation as a theoretical reference system.¹⁶⁰ This alone gave rise to ardent resistance as expressed by Capellen (1905) and others who held that, from a ‘monistic’ perspective, there was but one ‘natural’ or logically possible way to perceive major and minor chords and other sonorities, namely from bottom upwards. From a phenomenological point of view, the position of the (ideal) listener in a two-dimensional tonal space has been identified with the base note of the (major and minor) triad. The argument is that this position is implied by the structural composition of triads (thirds piled up on a base note acting as an anchor) as well as by our listening experience.¹⁶¹ Such assertions of course can be, and should be tested empirically.

In regard to ‘bottom-up’ vs. ‘top-down’ perception of vertical chordal structures, it has often been argued that the bottom-up adjustment would also apply to triads in a minor key, in particular if such are in root-position. According to a prediction model, perceptual roots of root-position diatonic triads in a minor key lie in the bass.¹⁶² Parncutt points to investigations of Bach chorales,¹⁶³ though, where it was found that the soprano voice is doubled more often than the bass.¹⁶⁴ Hence, “the soprano voice is a root candidate by virtue of its voicing alone, consistent with the idea that the soprano and the bass are the most perceptually salient voices in a four-part texture (presumably due to relative lack of masking by other voices).”

The ‘bottom-up’ orientation in regard to perception of chordal structures may in fact be not as ‘natural’ or ‘logical’ as it is for major chords in root position because of psycho-acoustic implications.¹⁶⁵ For example, in a (preliminary) experiment, twenty-one musically trained subjects were asked to judge four (complete or abridged) cadences played in minor keys on two variables: (a) harmonic coherence and (b) musical conclusiveness. The judgements were done on a scale from 1 = very low to 7 = very high. The cadences were taken from Riemann (1893) and other textbooks on harmony, and recorded with two synthesizers (Yamaha TX 81 Z) in just intonation tuning. Each cadence was presented (A) in original form, (B) with one voice added in the bass, and (C) one voice added on top of the soprano. To avoid marked shifts in spectral brightness, these additional voices were recorded -8 dB below the level of the main 4-part texture. The resulting sound pattern for cadence no. 1 is shown in figure 9.¹⁶⁶ The relative shift of the spectral centroid over time is documented in figure 10.

160 See Riemann 1895, 1905, 1914–16.

161 Albersheim 1979, 105/06 and 140.

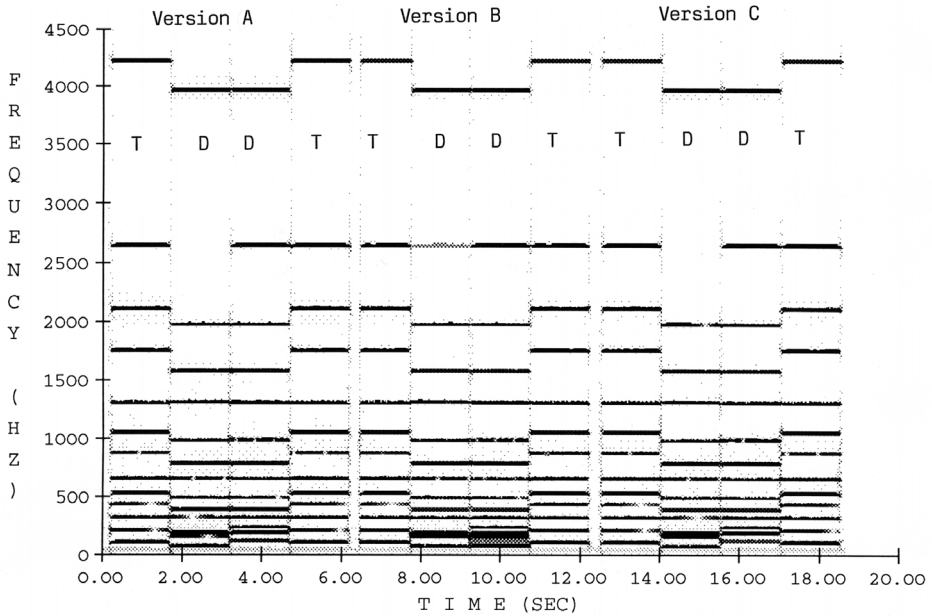
162 Parncutt 1997, 191.

163 Carried out by McHose 1947.

164 Parncutt 1997, 197.

165 See above in this text.

166 Adapted from Riemann 1893, 31, no. 30d/e.



file /me/Kadenz1.pv.an TX81Z a mf base freq = 27.50 Hz, thresh = 0.030

Figure 9. Cadence °T-°D, °D-°T (a-minor = e-Unterklang), three versions A, B, C

From figure 10 it is evident that adding a voice in the bass even at a reduced level adds to the ‘bottom end’ of the chord acoustically; moreover, the bass part is reinforced musically. This might lead to subjects’ judgement expressing at least a relative preference of this version (B) over the other two (A, C). The actual ratings, however, did not exhibit significant differences for the three versions:¹⁶⁷

Cadence 1	Sample file	Arithm. mean (a, b)	Stand. dev. (a, b)	Median (a, b)
Version A (orig.)	1, 2	4.48, 3.86	1.40, 1.35	4, 4
Version B (bottom)	3, 4	4.62, 3.81	1.50, 1.36	5, 3
Version C (top)	5, 6	4.57, 3.95	1.16, 1.20	4, 4

Table 3. Descriptors, cadence 1, (a) harmonic coherence, (b) musical conclusiveness, n = 21

Correlations between data (samples/files 1–6) show that the judgements are consistent within and between the two variables (r_{xy} ranges from $r_{24} = 0.472$, $p = 0.031$ to $r_{13} = 0.781$,

¹⁶⁷ The data, which are between ordinal and interval scale in quality, are processed as interval scaled. Data in all six samples have been checked for normal distribution and ‘outliers’ (Cusum test). A more detailed discussion of empirical data must be given elsewhere.

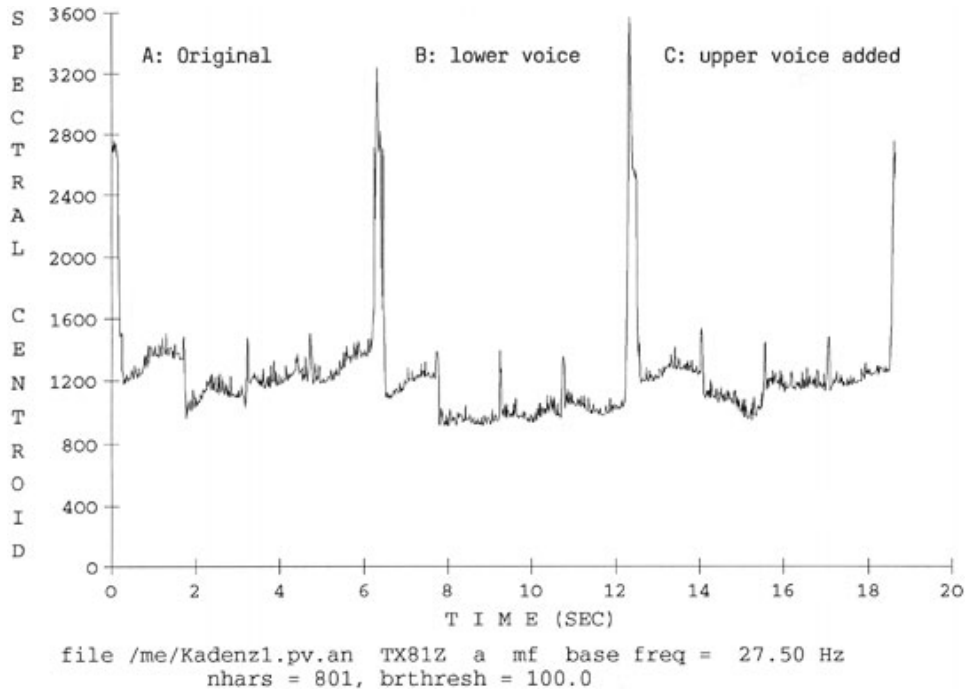


Figure 10. Spectral centroid./time of cadence (versions A, B, C)

$p < 0.001$). The only correlation that is not significant is r_{26} for musical conclusiveness between the original version (A) and version (C). After homogeneity of variance between samples had been checked with Bartlett-tests, ANOVA was performed where the versions A-C were taken as treatments of the factor 'cadence.' ANOVA gave no significant results. Neither did the six mean values (aM) subjected to a multiple-mean test (Scheffé) give any significant contrasts. Hence, the three versions seem to appear quite similar to listeners. Interestingly, version (B), which gained a little in harmonic coherence by adding a voice to the bottom, loses as much against the other two versions in conclusiveness.

Taking the data from five treatments, average ratings in general increased from the original (A) to versions (B, C) on both variables when either an additional voice below the bass or above the soprano were offered. Also, ratings increased when in one complete I-IV-I-V-I cadence a supporting voice was added, in the same sound stimulus, below the bass and another one above the soprano voice.

These findings, restricted as they are, at least do not support the assumption that listening attitude in regard to chords and chord progressions is always and invariably 'bottom-up' and centred at the bass. This is not to say that Oettingen's 'top-down' approach to 'Unterklänge' would duly apply. Rather, depending on musical context and, evidently, on the musical expertise of a listener trained to work out advanced 'Tonvorstellungen,'

chord progressions can be apprehended in ways incorporating ‘harmonic dualism.’¹⁶⁸ As with many things, this is a matter of training and experience.¹⁶⁹

The problem with ‘harmonic dualism’ is not that certain chords are conceived as ‘Unterklänge’ (as such in fact can be found in works of Wagner and other composers). A serious problem seems to be that, whereas in harmonic major the base note (Grundton) of a chord in ‘root position’ is also the note from where all tonal relations are the most simple ones (4:5:6), in harmonic minor the fifth must be chosen as ‘anchor’ (Bezugston) to derive the same simple proportions implied by Riemann’s ‘Ökonomie des Hörens.’¹⁷⁰ Consequently, a simple strategy of perceiving the ‘Grundton’ also as a cognitive anchor point would be missed in harmonic minor. In certain works, or parts thereof, listeners would be forced, in a strict dualistic approach, to shift between anchor points identified with the base note (for major chords), and the fifth (for minor chords). Consider, for example, bars 31–34 of Bach’s *Fantasia et Fuga in g* (BWV 542). The chord progression is as follows (uppercase letters = Major chords; lowercase letters = minor chords):

D-g-G-c-C-f-F-b^b-B^b-e^b-E^b-a^b-A^b-d^b-D^b- (diminished chords, bar 35) -e (bar 36)

In this passage, Bach not only moves around the ‘Circle of Fifths’ (that had been given a formal description in Heinichen 1728), from D to D^b, but alternates regularly between major and minor chords of the same key. Viewed from a perspective of strict dualism, the consequence would be to change the ‘anchor note’ (base note, fifth) and the direction of perceiving chords (upwards in major, downwards in minor) as indicated above. This is what the ‘monists’ blamed as ‘unnatural’ and contradicting common experience.¹⁷¹ The problem that a ‘Grundton’ (lowest note as played and heard) and a ‘Bezugston’ (center note of a chord defined by chord structure and harmonic function relevant to apprehension) might dissociate is not confined to harmonic minor, though, since inversions of major chords can bring about the same effect.

The degree of differentiation and exactitude musically trained subjects may achieve in regard to apprehending and conceptualizing tonal relations, of course varies according to certain objective and subjective factors. Apparently, advanced conceptualizations of intricate chord and key relationships as are found in many works of the 19th and early 20th century, can be developed by intensive training including listening, performance, and analysis. This is what Riemann understood by ‘Musikalische Logik,’ and what he still proposed in the *Ideen*. His own work, impressive as it is, would certainly have benefited

168 For case studies, see Vogel 1984, Bister 1994.

169 The author vividly remembers many sessions spent as a student at Bonn’s Musikwissenschaftliches Seminar where, in Martin Vogel’s classes devoted to harmonic analysis of works from Dufay to Debussy, our senior fellow student Heribert Bister would play parts from works (mostly Bach, Mozart, Chopin, Bruckner) on a difficult to master 72-note per octave ‘Reinharmonium’ (see Vogel 1975, 330–333; 1993, 345–348). He wanted to demonstrate, by means of different intonation patterns, several alternative solutions to harmonic problems posed by complex chord progressions (see Bister 1988). Such exercises (which followed suit to the work of Helmholtz, Oettingen and Riemann) helped with no doubt to develop our ‘Tonvorstellungen’.

170 See Vogel 1984, 25–27.

171 Perhaps most of all, Capellen 1905.

from empirical investigations relevant to music perception and cognition. Riemann was lacking support from psychologists and acousticians, and he may have been too idiosyncratic to cooperate in a team (should such an opportunity have existed). Oettingen, by contrast, notwithstanding his strong theoretical claims, was more of an empiricist,¹⁷² and carried out experiments on perception of major and minor chords (including aspects of tuning and intonation¹⁷³). This, however, was only possible in the last years of his life (after he had access to a special 'Reinharmonium' making different intonation patterns possible). Systematic research in areas of perception and cognition relevant to music theory remained scarce for decades to come.

4. Conclusion: Theory, 'Handwerkslehre,' Empirical Research

Music theory, since antiquity, combined rational analysis and speculation (as expressed by the term 'theoria') with empirical investigations. This line of tradition can be traced well into the 20th century¹⁷⁴ up to the present.¹⁷⁵ Already in the Renaissance and Baroque era, and parallel to more 'scientific' approaches to music theory (accentuating mathematical, acoustical and psychological aspects), description of musical structures with the goal of understanding as well as regulating actual composition gained ground. Whereas Zarlino (1558) strived to contribute to both areas, many later writers were concerned with fundamentals of 'Tonsetzkunst,' in the first place.¹⁷⁶ The shift of focus becomes obvious in works like Weber's *Versuch* where, at the end of the introductory chapter that still offers some elementary musical acoustics, the author leaves no doubt that his main subject is "Theorie der Tonsetzkunst oder Tonsatzlehre"¹⁷⁷ needed to prepare for the "Practische Ausübung der Tonsetzkunst," that is, musical composition.

'Tonsatzlehre' is what in fact is covered by 'practical' (or applied) music theory in many academic institutions. This field has been combined with music analysis in order to foster understanding of techniques of composition that were of importance in the past. Parallel and somewhat unconnected to such more or less 'practical' approaches, music theory in the 20th century saw a broad range of theorizing often based on abstract (mathematical, logical, computational) considerations and models, covering such diverse things as set theory, information theory, statistical distributions and probability functions, fuzzy logic, etc. By about 1970, a reorientation took place (more or less parallel with the so-called 'cognitive turn' in psychology and adaptation of concepts from Gestalt psychology) in which also structural linguistics played an important part. After serialism, on the one hand, and aleatoric as well as similar 'stochastic' concepts, on the other, had failed to attract listeners in significant quantities, a new interest in foundations

172 See Oettingen 1916.

173 See Wicke 1926.

174 See for example, Fokker 1945, Vogel 1975, 1993.

175 For example, Lerdahl 2001.

176 For example, Heinichen 1728, Kirnberger 1776.

177 Weber 1830, Vol. I, 17.

of tonal music viewed from a cognitive perspective awoke.¹⁷⁸ This perspective included experimental investigations as well as modelling of perceptual and cognitive processes, for example, by neural networks.¹⁷⁹ Only to sum up most of the relevant research that has been done in this area would require another article. It is enough, at this point, to indicate that there are many specialized publications reporting findings on specific problems (for example, concepts of ‘similarity’ as are relevant to music perception¹⁸⁰).

Taking into account developments of the past decades, one can say that nowadays basically several camps coexist (and mingle with each other, at least to some extent), one devoted to historical and applied ‘Tonsatzlehre’ (either ‘practically’ inclined or with a focus on certain eras and styles), another anchored in conventional areas of music theory (with a Schenkerian or, growing in the 1990s, “Neo-Riemannian” orientation). Then there are music psychologists with an interest in music theory as well as theorists who contribute to the psychology of music,¹⁸¹ not to forget investigations the neurosciences have done in music perception and cognition that also have implications for music theory.¹⁸² In addition, one could point to artistic concepts (some of them again backed by scientific orientations, for instance, in computer music) proposed by composers or ‘media artists’ working in or with sound.

Given this broad range of directions, research activities, and methodologies, it may be difficult to determine a single field of ‘music theory.’ What can be said, though, is that major areas traditionally considered to make up the core of tonal music theory (harmony, melody, rhythm and meter, form structure) have been subjected more and more to empirical investigations, to the effect that music theory nowadays seems to be ‘rule-based’ rather than ‘dogmatic,’¹⁸³ whereby ‘rule-based’ refers to perceptual and cognitive processes actually observed, or assumed to exist in expert listeners.¹⁸⁴ All in all, empirical studies have been given much more weight over the past decades. At the same time, theoretical concepts from the past (like ‘harmonic dualism’ as well as concepts of *Tonnetz* and ‘tonal space’ as explored by Oettingen, Fokker, Vogel, etc.) have been revitalized, and given new interpretations.¹⁸⁵ One of the tasks of systematic musicology is to contribute to empirical research relevant to substantiate music theory; another is to examine theories, models and empirical findings as they become available from various areas of research with the objective to seek coherent and causal explanations of musical phenomena.

178 For example, Fay 1971, 1974; Lerdahl and Jackendorff 1983. See also Krumhansl 1983, 1990; Seifert 1993.

179 See Leman 1995 and contributions in Leman 1997.

180 See contributions in Deliège 2007.

181 For example, Bruhn 1988, Eberlein and Fricke 1992, Ahlbäck 2004, Clarke 2005, Lerdahl and Krumhansl 2007.

182 For example, Patel 2008, Neuhaus 2008.

183 Dogmatic in a philosophical or other theoretical context means that a body of theory is regarded as authoritative and valid. See Rexroth 1971.

184 See Lerdahl and Jackendorff 1983, Narmour 1990, 1992.

185 See, for example, Harrison 1994, Lerdahl 2001.

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